

Contents

Preface	xiii
INTRODUCTION	xv
The place of the axiomatic approach in quantum field theory (xv). The layout of this book (xviii).	
Part I	
ELEMENTS OF FUNCTIONAL ANALYSIS AND THE THEORY OF FUNCTIONS	1
Synopsis	1
CHAPTER 1. Preliminaries on Functional Analysis	3
1.1. Normed Spaces	3
A. Linear spaces (3). B. Direct sum and tensor product of linear spaces (5). C. Normed spaces (7). D. Hilbert spaces (8). E. Direct sum and tensor product of Hilbert spaces (12). F. Linear functionals and dual spaces (14).	
1.2. Locally Convex Spaces	16
A. Equivalent systems of seminorms. Structure of LCS's (16). B. Fréchet Spaces (17). C. Examples (18).	
1.3. Linear Operators and Linear Functionals in Fréchet Spaces	20
A. Continuous maps of LCS's (20). B. The uniform boundedness principle. The weak and weak* topologies (22). C. The closed graph and open mapping theorems (23).	
1.4. Operators in Hilbert Space	25
A. The notion of an (unbounded) self-adjoint operator (25). B. Isometric, unitary and anti-unitary operators (28). C. The spectral theory of self-adjoint and unitary operators (29).	
1.5. Algebras with Involution. C^* -Algebras	31
A. Definition and elementary properties (31). B. The spectrum (33). C. Positive functionals (34). D. Representations (36). E. Trace class operators (41). F. Von Neumann algebras (43).	
CHAPTER 2. The Technique of Generalized Functions	46
2.1. The Concept of a Generalized Function	46
A. Functional definition (46). B. Definition in terms of fundamental sequences (49). C. Local properties of generalized functions (51).	
2.2. Transformation of Arguments and Differentiation	53
A. Change of variables in a generalized function (53). B. Differentiation of generalized functions. Examples (54).	
2.3. Multiplication of a Generalized Function by a Smooth Function	56
A. The problem underlying multiplication of generalized functions. The concept of a multiplicator (56). B. The division problem (58).	
2.4. The Kernel Theorem. Tensor Products of Generalized Functions	61
A. Bilinear functionals on spaces of type S (61). B. Tensor products (62).	

CONTENTS

2.5. Fourier Transform and Convolution	63
A. Fourier transform of test functions (63). B. Fourier transform of generalized functions (65). C. Convolutes (66). D. Generalized functions of integrable type (67). E. Convolution of generalized functions (70).	
2.6. Generalized Functions Dependent on a Parameter	72
A. General information (72). B. Restriction of generalized functions (74). C. More on the multiplication of generalized functions (76).	
2.7. Vector- and Operator-Valued Generalized Functions	78
A. Generalized functions with values in Hilbert space (78). B. Operator-valued generalized functions (80). C. The notion of a generalized eigenvector (82).	
Appendix A. Generalized Functions on Subsets of \mathbf{R}^n	83
A.1. Generalized functions on an open subset (83). A.2. Generalized functions on canonically closed regular subsets (84). A.3. Application: generalized functions on the compactified sets $[A, \infty]$, \mathbf{R}_∞ , $[-\infty, +\infty]$ (86).	
Appendix B. The Laplace Transform of Generalized Functions	89
B.1. The Laplace transform as an analytic function in the complex plane (89). B.2. The case of a generalized function with support in a pointed cone (96). B.3. Example: generalized functions of retarded type (98). B.4. Boundary values of the Laplace transform (99). B.5. Example: the “mathematics” of dispersion relations (103). B.6. Restriction of the Laplace transform (105).	
Appendix C. Homogeneous Generalized Functions	106
C.1. Homogeneous generalized functions in $\overset{\circ}{\mathbf{R}}{}^n$ (106). C.2. The single real variable case (109). C.3. Extension of homogeneous generalized functions (110). C.4. Application to covariant homogeneous generalized functions (113). C.5. Homogeneous generalized functions in the complex plane (114).	
CHAPTER 3. Lorentz-Covariant Generalized Functions	118
3.1. The Lorentz Group	118
A. The geometry of Minkowski space (118). B. Definition of the general Lorentz group and its connected components (119). C. The universal covering of the group L_+^\uparrow (121). D. Finite-dimensional representations of the group $SL(2, C)$ (125). E. Simply reducible finite-dimensional representations of $SL(2, C)$. Spatial reflection (128).	
3.2. Lorentz-Invariant Generalized Functions in Minkowski Space	131
A. Definition (131). B. Even invariant generalized functions. Invariant generalized functions with support at a point (132). C. Odd invariant generalized functions (136).	
3.3. Lorentz-Covariant Generalized Functions in Minkowski Space	138
A. Definition (138). B. Structure of covariant generalized functions (139).	
3.4. The Case of Several Vector Variables	143
A. Generalized functions that are invariant with respect to a compact group (143). B. Generalized functions that are covariant with respect to a compact group (149). C. Applications to Lorentz-invariant and Lorentz-covariant generalized functions (155).	
Appendix D. Vocabulary of Lie Groups and their Representations	159

D.1. Abstract groups. Algebraic properties (159).	D.2. Lie groups (160).
D.3. Lie algebras (162).	D.4. Relation between Lie groups and Lie algebras (163).
D.5. Local Lie groups. Canonical parametrization.	Lie's theorems (164).
D.6. Linear representations (166).	D.7. Adjoint and co-adjoint representations. Killing forms (167).
CHAPTER 4. The Jost-Lehmann-Dyson Representation	
4.1. Relation between the JLD Representation and the Wave Equation	170
A. Preliminary remarks (170).	B. Outline of the derivation (171).
C. Departure into six-dimensional space (173).	
4.2. Properties of Solutions of the d'Alembert Equation in S'	175
A. Notation (175).	B. Fundamental Solution of the Cauchy Problem (176).
C. Cauchy problem on a spacelike hypersurface; Huygens' principle (179).	
D. The Asgeirsson formula and its applications (183).	
4.3. Derivation of the Jost-Lehmann-Dyson Formula	185
A. Construction of the spectral function (185).	B. Further properties of the support of the spectral function (188).
C. Examples (192).	D. Representations for generalized functions of retarded and advanced types (193).
CHAPTER 5. Analytic Functions of Several Complex Variables	
5.1. Properties of Holomorphic Functions. Plurisubharmonic Functions	197
A. Space of holomorphic functions (197).	B. Holomorphy and analyticity (199).
C. Analytic continuation (200).	D. Generalized principle of analytic continuation; "edge of the wedge" theorem (204).
E. Holomorphic distributions (207).	F. Invariant and covariant analytic functions (209).
G. Plurisubharmonic functions (211).	
5.2. Domains of Holomorphy	215
A. Holomorphic convexity (215).	B. Pseudo-convexity (217).
C. Modified principle of continuity (219).	D. Single-sheeted envelopes of holomorphy (221).
E. Invariant domains (223).	F. An example of holomorphic extension (226).
Part II	
RELATIVISTIC QUANTUM SYSTEMS	231
Synopsis	231
CHAPTER 6. Algebra of Observables and State Space	
6.1. Algebraic Formulation of Quantum Theory	233
A. Algebra of observables. States (233).	B. Transition probability (235).
C. Relationship to representations (236).	
6.2. Superselection Rules	239
A. The role of pure vector states (239).	B. Standard superselection rules (243).
C. Connection with gauge groups (245).	D. Example of non-abelian gauge groups (247).
6.3. Symmetries in the Algebraic Approach	249
A. The concept of symmetry (249).	B. Proof and discussion of Wigner's theorem (252).
C. Symmetry groups (256).	

6.4. Canonical Commutation Relations	260
A. The role of the Schrödinger representation (260). B. Infinite number of degrees of freedom (263). C. Proof of von Neumann's uniqueness theorem (267).	
CHAPTER 7. Relativistic Invariance in Quantum Theory	270
7.1. The Poincaré Group	270
A. Definition (270). B. Reflections (271). C. The Lie algebra of the Poincaré group (272).	
7.2. Unitary Representations of the Proper Poincaré Group	274
A. Poincaré invariance condition (274). B. Classification of irreducible representations of ρ_0 . Spectral principle (275). C. Description of representations corresponding to particles with positive mass (280). D. Manifestly covariant realization of "physical" irreducible representations (284).	
7.3. Fock Space of Relativistic Particles	288
A. Second quantization space (288). B. Connection with (anti-)commutation relations (292). C. Covariant creation and annihilation operators (296). D. Symmetries of the general Poincaré group (299). E. Relativistic scattering matrix (302). F. Kinematics of two-particle processes (307).	
Appendix E. Four-Component Spinors and the Dirac Equation	310
E.1. Clifford algebra over Minkowski space (310). E.2. Spinor representation of the Lorentz group; various realizations of the γ -matrices (312). E.3. Dirac equation; representations of the Poincaré group with spin 1/2 (314).	
Part III	
LOCAL QUANTUM FIELDS AND WIGHTMAN FUNCTIONS	318
Synopsis	318
CHAPTER 8. The Wightman Formalism	321
8.1. Quantum Field Systems	321
A. Concept of localization (321). B. Principle of local commutativity (322). C. "Fundamental" fields and "physical" fields (323).	
8.2. Definition and Properties of a Local Quantum Field	324
A. Wightman's axioms (324). B. Discussion of the axioms (325). C. Irreducibility of fields (329). D. Separating property of the vacuum vector (331).	
8.3. Wightman Functions	332
A. Characteristic properties of Wightman functions (332). B. Källén-Lehmann representation for a scalar field (335). C. Reconstruction of the theory from the Wightman functional (337).	
8.4. Examples: Free and Generalized Free Fields	340
A. Free scalar neutral field (340). B. Free scalar charged field (345). C. Free Dirac field (348). D. Generalized free fields (351).	
Appendix F. Summary of Invariant Solutions and Green's Functions of the Klein-Gordon Equation	353
Appendix G. General Form of the Covariant Two-Point Function	355
G.1. Covariant decompositions compatible with locality (355). G.2. Decomposition with respect to spin (356).	

CHAPTER 9. Analytic Properties of Wightman Functions in Coordinate Space	359
9.1. Bargmann-Hall-Wightman Theorem and its Corollaries	359
A. Complex Lorentz transformations (359). B. Lorentz-covariant analytic functions in the past tube (362). C. Real points of the extended tube (366). D. Analyticity of Wightman functions in a symmetrized tube (368). E. Global nature of locality (371).	
9.2. <i>TCP</i> -Theorem	375
A. <i>TCP</i> -invariance (375). B. Weak locality (378). C. Borchers classes; the notion of a local composite field (378).	
9.3. Connection between Spin and Statistics	381
A. Statement of the results (381). B. Necessary conditions for anomalous commutation relations (383). C. Reduction of ω to canonical form (385). D. Construction of the Klein transformation (387).	
9.4. Equal-Time Commutation Relations. Haag's Theorem	388
A. Three-dimensional version of Haag's theorem (388). B. Haag's theorem in the relativistic theory (391). Comments on Haag's theorem (392).	
9.5. Euclidean Green's Functions	394
A. Group of rotations of four-dimensional Euclidean space (394). B. Properties of the Schwinger functions (396). C. Reconstruction theorem in terms of Schwinger functions (400).	
Appendix H. Parastatistics	403
H.1. Free parafields and paracommutation relations (403). H.2. Comment on the <i>TCP</i> -theorem and the connection between spin and parastatistics for local parafields (406).	
Appendix I. Infinite-Component Fields	407
I.1. Elementary representation of $SL(2, C)$ (407). I.2. Concept of a quantum ICF (408). I.3. Covariant structure of the two-point function. Infinite degeneracy of mass with respect to spin (410). I.4. Absence of \mathfrak{P}_+ -covariance and connection between spin and statistics in ICF models (413).	
CHAPTER 10. Fields in an Indefinite Metric	417
10.1. Pseudo-Wightman Formalism	417
A. Pseudo-Hilbert space (417). B. Axioms of pseudo-Wightman type (420). C. Vacuum sector and charged states (423). D. Physical subspace of pseudo-Hilbert space (427).	
10.2. Abelian Models with Gauge Invariance of the 2nd Kind	428
A. The field of the dipole ghost and the gradient model (428). B. Local formulation of quantum electrodynamics (434).	
10.3. Internal Symmetries	440
A. Symmetries and currents in the Wightman formalism (440). B. Goldstone's theorem (443). C. Spontaneous symmetry breaking in abelian gauge theories (446).	
CHAPTER 11. Examples: Explicitly Soluble Two-Dimensional Models	450
11.1. Free Scalar Massless Field in Two-Dimensional Space-Time	450
A. One-dimensional non-canonical scalar field (450). B. Physical representation (454). C. Free "quark" fields; bosonization of fermions (461). D. Free scalar massless "ghost" field (467).	

CONTENTS

11.2. The Thirring Model	469
A. Solution of the field equation (469). B. Currents and charges; vacuum representation (473).	
11.3. The Schwinger Model	474
A. Solution in the Lorentz gauge (474). B. Vacuum functional (480). C. Physical fields; observables (481).	
Part IV	
COLLISION THEORY. AXIOMATIC THEORY OF THE <i>S</i> -MATRIX	484
Synopsis	484
CHAPTER 12. Haag-Ruelle Scattering Theory	486
12.1. Scheme of the Quantum Field Theory of Scattering	486
A. The one-particle problem in quantum field theory (486). B. Construction of in- and out-states (488). C. <i>S</i> -matrix and <i>TCP</i> -operators in the asymptotically complete theory (489).	
12.2. Existence of Asymptotic States	491
A. Truncated vacuum expectation values (491). B. Strengthened cluster property (495). C. Spread of relativistic wave packets (497). D. Proof of the main result (501).	
CHAPTER 13. Lehmann-Symanzik-Zimmermann Formalism	503
13.1. Basic Concepts	503
A. <i>T</i> -products of fields (503). B. Retarded products (509). C. LSZ axioms (512).	
13.2. Asymptotic Conditions and Reduction Formulae	515
A. LSZ asymptotic conditions (515). B. Yang-Feldman equations (520). C. Partial reduction formulae (522). D. Reduction formulae for the scattering matrix (526).	
CHAPTER 14. The <i>S</i>-Matrix Method	530
14.1. <i>S</i> -Matrix Formulation of the Basic Requirements of the Local Theory . .	530
A. The concept of extending the <i>S</i> -matrix beyond the mass shell (530). B. Choice of the class of test functions (534). C. Axioms of the <i>S</i> -matrix approach (535). D. Radiation operators; current (537).	
14.2. Fields in the Asymptotic Representation	540
A. Construction of quantum fields and their <i>T</i> -products (540). B. Fulfillment of the LSZ axioms (544).	
Part V	
CAUSALITY AND THE SPECTRAL PROPERTY: THE ORIGINS OF THE ANALYTIC PROPERTIES OF THE SCATTERING AMPLITUDE	546
Synopsis	546
CHAPTER 15. Analyticity with respect to Momentum Transfer and Dispersion Relations	548
15.1. The Lehmann Small Ellipse	548
A. Introductory remarks (548). B. JLD representation for retarded and advanced (anti)commutators (551). C. Analyticity with respect to <i>t</i> (553).	

15.2. Dispersion Relations	556
A. The main steps for the derivation of the dispersion relations (556). B. Passage to the complex domain with respect to the momenta p_2, p_4 (557). C. Dispersion relation for non-physical “masses” (560). D. Analytic properties of the absorptive part of the amplitude (563). E. Dispersion relation on the mass shell (570).	
CHAPTER 16. Analytic Properties of the Four-Point Green’s Function	574
16.1. Generalized Retarded Functions	574
A. Generalized retarded products (574). B. Supports in x -space (576).	
16.2. Four-Point Green’s Functions	579
A. Notation (579). B. Domains of coincidence in p -space (581). C. Steinmann identities (583). D. Analyticity near physical points (586).	
16.3. Crossing Relation	589
A. Statement of the result (589). B. The case of imaginary “masses” (590). C. Analytic continuation with respect to the mass variables (591). D. Passage to the mass shell (596).	
Appendix J. The Role of Unitarity	598
J.1. Partial wave decomposition of the scattering amplitude of a two-particle process (598). J.2. Analytic continuation of the dispersion relation with respect to t (603).	
CHAPTER 17. Consequences for High-Energy Elementary Processes	608
17.1. Restrictions on the Behaviour of Cross Sections at High Energies	608
A. Froissart bound (608). B. Comparison of the cross sections of the interaction of a particle and an antiparticle with the same target (613).	
17.2. Inclusive Processes	617
A. Physical characteristics of inclusive processes (617). B. Analytic properties of differential cross sections with respect to angular variables (621). C. Asymptotic estimates (624).	
Commentary on the Bibliography and References	626
Bibliography	644
References	649
Index of Notation	683
Index	687