Contents

CHAPTER I QUANTUM MECHANICS OF THE PHOTON

§ 1.	The Photon Wave Function	1
	1. Introduction. 2. The Photon Wave Function in \mathbf{k} -Space. 3. Energy. 4. Normalization of the Photon Wave Function.	
§ 2.	Photon States of Definite Momentum	9
	1. Photon Momentum Operator. 2. Impossibility of In- troducing a Photon Wave Function in the Coordinate Representation. 3. Plane Waves. 4. Polarization Density Matrix for the Photon.	
§ 3.	Angular Momentum. Photon Spin	17
	1. Angular Momentum Operator. 2. Photon Spin Oper- ator. 3. Photon Spin Wave Functions.	
§4.	Photon States of Definite Angular Momentum and	
	Parity	24
	1. Eigenfunctions of the Photon Angular Momentum Operator. 2. Longitudinal and Transverse Vector Spher- ical Harmonics. 3. Parity of Photon States. 4. Expan- sion in Spherical Waves. 5. Expressions for the Electric and Magnetic Fields.	
§ 5.	Scattering of Photons by a System of Charges	36
	1. Incoming and Outgoing Waves. 2. Effective Scattering Cross Section. 3. The Optical Theorem. 4. Dispersion Relations.	
§ 6.	The Photon Field Potentials	46
	1. Transverse, Longitudinal, and Scalar Potentials. 2. Longitudinally Polarized "Photon." 3. Potentials for	
	Plane and Spherical Waves.	

x	CONTE	ENTS
§ 7.	 System of Photons 1. Wave Function for a System of Two Photons. 2. Even and Odd States of Two Photons. 3. Classification of the States of Two Photons of Definite Angular Momentum. 4. Wave Function for a System of an Arbitrary Number of Photons. 	52
§ 8.	L-Vectors and Spherical Harmonics 1. Irreducible Tensors. 2. The Algebra of L-Vectors. 3. Spherical Harmonics. CHAPTER II RELATIVISTIC QUANTUM MECHANICS OF THE ELECTRON	62
§ 9.	 The Dirac Equation 1. Spinors. Pauli Matrices. 2. Dirac Equations. Dirac Matrices. 3. Unitary Transformations of Bispinors. 4. The Necessity for Four-Component Electron Wave Functions. 5. Symmetric Form of the Dirac Equation. Equation of Continuity. 6. Invariance of the Dirac Equation. 7. Bilinear Combinations of the Components of the Wave Function. 	73
§ 10.	 Electron and Positron States. States of Definite Momentum and Polarization Solutions with Positive and Negative Frequencies. The Charge Conjugation Transformation. The Positron Wave Function. Plane Waves. Polarization Density Matrix for the Electron. Averaging over Polarization States. 	86
§ 11.	Electron States of Definite Angular Momentum and Parity 1. Orbital and Spin Functions. Spherical Spinors. 2. Wave Function of a State of Definite Angular Momentum. 3. Parity of a State. 4. Expansion in Spherical Waves.	105
§ 12.	Electron in an External Field 1. The Dirac Equation with an External Field. 2. Sepa- ration of Variables in a Central Field. 3. Asymptotic Be- havior of the Radial Functions. 4. Behavior of Energy Levels as Functions of the Potential Well Depth. 5. Elec- tron in a Constant Homogeneous Magnetic Field.	112

§ 13.	 Motion of an Electron in the Field of a Nucleus 1. Solution of the Radial Equations for the Coulomb Field. 2. Wave Functions for the Continuous Spectrum. 3. Isotopic Level Shift. 4. General Investigation of the Effect of Finite Nuclear Size. 	122
§ 14.	 Electron Scattering 1. Spinor Scattering Amplitude. 2. Expression for the Cross Section in Terms of Phases. 3. Polarization and Azimuthal Asymmetry. 4. Scattering by a Coulomb Field. 5. Small Angle Scattering. 	131
§ 15.	 Nonrelativistic Approximation 1. Transition to the Pauli Equation. 2. Second Approximation. 3. Application of the Dirac Equation to Nucleons. 	144
	CHAPTER III	
	QUANTIZED ELECTROMAGNETIC AND ELECTRON-POSITRON FIELDS	
§ 16.	 Quantization of the Electromagnetic Field 1. Four-Dimensional Form of the Field Equations. 2. Variational Principle. Energy-Momentum Tensor of the Electromagnetic Field. 3. Expansion of the Potentials into Plane Waves. 4. Quantization of the Electromagnetic Field. 5. Use of the Indefinite Metric. 	153
§ 17.	Commutators of the Electromagnetic Field 1. Commutation Relations for the Potentials and the Field Components. 2. Chronological and Normal Products of Components of the Potential. 3. Singular Functions As- sociated with the Operators \Box and $(\Box - m^2)$.	174
§ 18.	 Quantization of the Electron-Positron Field 1. Variational Principle for the Dirac Equation. Energy- Momentum Tensor of the Electron-Positron Field. 2. Quantization Rules for the Electron-Positron Field. 	195
§ 19.	Anticommutators of the Electron-Positron Field. Chronological and Normal Products of Field Compo- nents. Current Density 1. Commutation Relations for Field Components. 2. Chronological and Normal Products of Operators of	205

22

4

the Electron-Positron Field. 3. Electric Current Density.

xi

214

§ 20. General Properties of Wave Fields .

 Wave Functions of a Field and the Lorentz Group.
 Irreducible Finite-Dimensional Representations of the Lorentz Group.
 Energy-Momentum Tensor and Angular Momentum Tensor.
 Current Density Vector.
 Relativistically Invariant Field Equations.
 Wave Equations for Particles of Spin Zero and Unity.

§ 21. Quantization of Fields. Connection between Spin and Statistics 237

 Nondefiniteness of the Charge in the Case of Integral Spin and of the Energy in the Case of Half-Integral Spin.
 Quantization of Fields for Integral and Half-Integral Spin. Pauli's Theorem. 3. Inversion of Coordinates and Time Reversal.

CHAPTER IV

FUNDAMENTAL EQUATIONS OF QUANTUM ELECTRODYNAMICS

§ 22. Interacting Electromagnetic and Electron-Positron Fields

253

1. System of Equations for Interacting Fields. 2. Lagrangian. Energy-Momentum Tensor. 3. Field Equations in Poisson Bracket Form. 4. Invariance Properties of the Equations of Quantum Electrodynamics.

§ 23. Equations of Quantum Electrodynamics in the Interaction Picture. Invariant Perturbation Theory 1. Heisenberg and Schrödinger Pictures. Interaction Pic-

268

Heisenberg and Schrödinger Pictures. Interaction Picture.
 Transition to the Interaction Picture in Quantum Electrodynamics.
 Charge Conjugation Operator.
 Perturbation Theory.

§ 24. The Scattering Matrix

290

1. The Scattering Problem and the Definition of the Scattering Matrix. 2. Matrix Elements of Field Operators. 3. Representation of the Scattering Matrix as a Sum of Normal Products. 4. General Relation between *T*-and *N*-Orderings. 5. Symmetry of the Scattering Matrix under Time Reversal.

xii

§ 27.

§ 25.	Graphical Representation of the Elements of the Scat- tering Matrix. The Scattering Matrix in Momentum Space					
	1. Graphical Representation of Normal Products. 2. Var- ious Interaction Processes between Fields. 3. Transition to Momentum Space. 4. Closed Electron Loops with an Odd Number of Vertices. 5. Rules for Writing Down Matrix Elements.					
§ 26.	Probabilities of Various Processes	327				
	1. General Formula for the Probability. 2. Effective Cross Section. 3. Summation and Averaging over Polari- zation States of Electrons and Photons. 4. Probabilities of Processes Involving Polarized Particles. 5. Probabili- ties of Processes in the Presence of an External Field. 6. Feynman's Notation.					
	CHAPTER V					
	INTERACTION OF ELECTRONS WITH PHOTONS					

Emission and Absorption of a Photon . . .

1. General Expression for the Matrix Element. 2. Electric Multipole Radiation. 3. Magnetic Multipole Radiation. 4. Selection Rules. 5. Angular Distribution and Polarization of the Radiation.

§ 28. Scattering of a Photon by a Free Electron 363

1. Scattering Matrix Element. 2. Application of Conservation Laws. 3. Differential Cross Section for Unpolarized Particles. 4. Angular Distribution and Total Cross Section. 5. Distribution of Recoil Electrons. 6. Scattering of Polarized Photons. 7. Scattering of Photons by Polarized Electrons.

§ 29. Bremsstrahlung

378

1. Perturbation Theory for an Electron Wave Function in the Continuum. Incoming and Outgoing Waves. 2. Effective Cross Section for Bremsstrahlung. 3. Angular Distribution of the Radiation in a Coulomb Field. 4. Polarization of the Radiation. 5. Spectrum of the Radiation. 7. Radiative Energy Losses. 8. Exact 6. Screening. Theory of Bremsstrahlung in the Nonrelativistic Domain. 9. Exact Theory of Bremsstrahlung in the Extreme Relaxiii

27

345

438

tivistic Domain. 10. Radiation Emitted in Electron-Electron and Electron-Positron Collisions.

- § 30. Emission of Photons of Long Wavelength 413
 1. "The Infrared Catastrophe." 2. Investigation of the Divergence in the Low Frequency Domain by Means of the Scattering Matrix. 3. Relation between the Photon "Mass" and the Minimum Frequency.
- § 31. Photoeffect 429
 1. Photoeffect in the Nonrelativistic Domain.
 2. Photoeffect in the Relativistic Domain.

§ 32. Production of Electron-Positron Pairs

1. Production of an Electron-Positron Pair by a Photon in the Field of a Nucleus. 2. Exact Theory of Pair Production by a Photon in the Field of a Nucleus in the Nonrelativistic and Extreme Relativistic Cases. 3. Pair Production by Two Photons. 4. Pair Production in a Photon-Electron Collision. 5. Pair Production in a Collision of Two Fast Charged Particles.

§ 33. Annihilation of Electron-Positron Pairs into Photons 457
1. Annihilation of a Pair into Two Photons. 2. Polarization Effects in the Two-Photon Annihilation of a Pair.
3. Annihilation of a Pair into One Photon. 4. Positronium Decay. 5. Three-Photon Decay of Orthopositronium. 6. Multiple Photon Production Accompanying the Annihilation of a Pair.

 The Number of Equivalent Photons. 2. Bremsstrahlung from a Fast Electron in the Field of a Nucleus.
 Radiation Emitted in an Electron-Electron Collision.
 Pair Production by a Photon in the Field of a Nucleus.
 Pair Production in a Collision of Two Fast Particles.

§ 35.	Scattering of a P	ho	ton by	a	Bound	Elec	tron.	Em	issic	m	
	of Two Photons		•								484
	1 The Dispersi	on	Form	ปล	2 R	eson	ance	Scat	terin	in a	

1. The Dispersion Formula. 2. Resonance Scattering. 3. Compton Scattering by Bound Electrons. 4. Emission of Two Photons. The Metastable $2s_{\frac{1}{2}}$ State of the Hydrogen Atom.

xiv

§ 34.

CHAPTER VI

RETARDED INTERACTION BETWEEN TWO CHARGES

§ 36.	Electron-Electron and Positron-Electron Scattering	499
	1. Electron-Electron Scattering. 2. Positron-Electron Scattering. 3. Scattering of Polarized Electrons and Positrons. 4. Annihilation of an Electron-Positron Pair into a μ -Meson Pair.	
§ 37.	Retarded Potentials	509
	1. Interaction Function for Two Charges. 2. General Form of the Matrix Element. 3. Retarded Potentials and Transition Currents.	
§ 38.	Interaction Energy of Two Electrons to Terms of Order v^2/c^2	517
	1. The Breit Formula. 2. Schrödinger Equation for a Two-Electron System. 3. Interaction between an Elec- tron and a Positron. 4. Exchange Interaction between an Electron and a Positron.	517
§ 39.	Positronium	527
	 Hamiltonian Operator and the Unperturbed Equation. Perturbation Operator. 3. Fine Structure. 4. Zeeman Effect. 	
§ 40.	Internal Conversion of Gamma-Rays	537
	 Expansion of Retarded Potentials in Spherical Waves. Conversion Coefficient. 3. Conversion in the K-Shell. Effect of Finite Nuclear Size. 5. Effect of Electron Shells on Radiation from the Nucleus. 	
§ 41.	Conversion Accompanied by Pair Production. Excita-	
	tion of Nuclei by Electrons	554
	tion of Nuclei by Electrons 1. Conversion of Magnetic Multipole Radiation. 2. Con- version of Electric Multipole Radiation. 3. Excitation of Nuclei by Electrons. 4. Monoenergetic Positrons.	554
§ 42.	 tion of Nuclei by Electrons 1. Conversion of Magnetic Multipole Radiation. 2. Conversion of Electric Multipole Radiation. 3. Excitation of Nuclei by Electrons. 4. Monoenergetic Positrons. Coulomb (Monopole) Transitions 	554 565

CHAPTER VII INVESTIGATION OF THE SCATTERING MATRIX § **43**. **Properties of Exact Solutions of the Equations of** 571 **Quantum Electrodynamics. Propagators** 1. Stationary States of a System of Interacting Fields. 2. Propagators and Their Spectral Representation. 3. Connection between Propagators and the Scattering Matrix. Integral Equations for Propagators. 4. Electromagnetic Mass of the Electron. **§ 44.** 593 Structure of the Scattering Matrix . 1. Self-Energy Parts of Diagrams. 2. Vertex Parts of Diagrams. 3. Renormalization of Electron Mass. § 45. **Renormalization of Electron Charge** 605 1. Physical Charge of the Electron. 2. Renormalization of Propagators and Vertex Parts. 3. Three-Photon Vertex Parts. 4. Renormalization of Matrix Elements. 5. Formulation of Perturbation Theory as an Expansion of Powers of e_c . § 46. **Divergences in the Scattering Matrix and their** Removal . . 619 . . . 1. Divergences in Irreducible Diagrams. 2. Introduction of a Cut-Off Momentum. 3. Convergence of Regularized Expressions for Irreducible Vertex Parts and Self-Energy Parts. 4. Convergence of Regularized Quantities in the Case of Reducible Diagrams. § 47. Evaluation of Self-Energy and Vertex Parts 631 1. Evaluation of Integrals over Four-Dimensional Re-2. Second Order Electron Self-Energy Part. gions. 3. Second Order Photon Self-Energy Part. 4. Third Order Vertex Part in the Case of External Electron Lines. 5. Third Order Vertex Part in the Case of One External Electron Line. § 48. **Functional Properties of Green's Functions.** Limits of Applicability of Quantum Electrodynamics . 657 1. Expansion Parameters of Perturbation Theory. 2. Zero Order Approximation in the Expansion in Powers of e^2 .

3. Integral Equations for the Zero Order Approximation.

§ 49.

4. The Renormalization Group. 5. Derivation of Asymptotic Expressions for the Green's Functions with the Aid of Differential Equations of the Renormalization Group. 6. The Problem of Closure of Quantum Electrodynamics.

676

1. Green's Functions in the Presence of External Fields. 2. Green's Function for Two Electrons. Equation for Bound States of the Electron-Positron System. 3. Equations for Green's Functions in Terms of Variational Derivatives. 4. Expressions for Green's Functions in Terms of Functional Integrals.

Generalized Green's Functions

CHAPTER VIII

RADIATION CORRECTIONS TO ELECTRO-MAGNETIC PROCESSES

§ 50. Effective Potential Energy of the Electron. Radiation Corrections to the Electron Magnetic Moment and to **Coulomb's Law** . . .

> 1. Energy of Interaction of the Electron with the Electromagnetic Field Taking into Account Corrections of Order α . 2. Radiation Corrections to the Electron Magnetic Moment. 3. Radiation Corrections to Coulomb's Law.

§ 51. **Radiation Corrections to Electron Scattering**

1. Electron Scattering by the Coulomb Field of a Nucleus in the Second Born Approximation. 2. Differential Cross Section for the Scattering of an Electron by the Coulomb Field of a Nucleus taking into Account Radiation Corrections of Order α . 3. Elimination of the Photon "Mass" from the Scattering Cross Section. 4. Removal of the Infrared Divergence for an Arbitrary Scattering Process. 5. Scattering of High Energy Electrons by an External Field. 6. Radiation Corrections to Electron-Electron and Electron-Positron Scattering.

§ 52. **Radiation Corrections to Photon-Electron Scattering**, to Pair Creation and Annihilation, and to Bremsstrahlung 731 1. Radiation Corrections to the Compton Effect. 2. Limiting Cases of Low and High Energies. 3. Radiation

705

693

Corrections to Two-Photon Pair Annihilations. 4. Radiation Corrections to Bremsstrahlung. 5. Radiation Corrections to Photon Production and Single Photon Annihilation of Pairs.

§ 53. Radiation Corrections to Atomic Levels 751 1. Radiation Shift of Atomic Levels. 2. Radiation Shift of the Levels of μ-Mesohydrogen. 3. Natural Line Widths. 4. Photon Scattering near Resonance.

§ 54. Photon-Photon Scattering and the Lagrangian for the Electromagnetic Field

764

1. Photon-Photon Scattering Tensor of the Fourth Rank. 2. Photon-Photon Scattering. 3. Connection between the Photon-Photon Scattering Cross Section and the Radiation Corrections to the Lagrangian of the Electromagnetic Field. 4. Exact Expressions for the Lagrangian of the Electromagnetic Field.

§ 55. Photon Scattering by the Coulomb Field of a Nucleus 792

1. General Expression for the Cross Section for Photon Scattering by a Constant Electromagnetic Field. 2. Relation between the Forward Scattering Amplitude for a Photon and Pair-Production by a Photon in the Field of a Nucleus. 3. Momentum Distribution of Recoil Nuclei Accompanying Pair Production by a Photon in the Field of a Nucleus. 4. Angular Distribution of Recoil Nuclei and Total Cross Section for Pair Production by a Photon in the Coulomb Field of a Nucleus. 5. Small Angle Coherent Scattering of Photons by the Field of a Nucleus.

CHAPTER IX

ELECTRODYNAMICS OF PARTICLES OF SPIN ZERO

- § 56. Field Equations for Scalar Particles
 819
 1. First Order Equations.
 2. Quantization of the Free Scalar Field.
 3. Commutators of the Field. Vacuum Expectation Values of Products of Field Components.
- § 57. The Scattering Matrix in Scalar Electrodynamics 827
 1. The Interaction Picture. 2. Rules for Calculating Elements of the Scattering Matrix. 3. Divergences of the Scattering Matrix.

xviii

§ 58.	Scattering of Scalar Particles 1. Scattering of Scalar Particles by the Coulomb Field of a Nucleus. 2. Scattering of a Charged Scalar Particle by a Scalar Particle.	835
§ 59.	Scattering of a Photon by a Scalar Particle. Brems- strahlung Photons from a Scalar Particle 1. Scattering of a Photon by a Scalar Particle. 2. Brems- strahlung from Scalar Particles.	838
§ 60. § 61.	 Production and Annihilation of Pairs of Scalar Particles 1. Production of Pairs of Scalar Particles by a Photon in the Coulomb Field of a Nucleus. 2. Production of a Pair of Scalar Particles by Two Photons. 3. Two-Photon An- nihilation of a Pair of Scalar Particles. 4. Annihilation of Pairs of Scalar Particles into Electron-Positron Pairs and the Inverse Process. Polarization of the Vacuum in the Case of Charged Scalar Particles 1. Vacuum Polarization Tensor for Scalar Particles. 2. Correction to Coulomb's Law. 3. Photon-Photon Scata 	842 847
	tering. Radiation Corrections to the Lagrangian of the Electromagnetic Field.	
Conclu	uding Remarks	852
Refere	ences	855
Subje	ct Index	863