

# CONTENTS

*Preface* *page* xix

*List of principal symbols* xxi

## PART I. FLUID MOTION THEORY

### *Chapter 1.* INTRODUCTORY THEORY

1.1 Introduction 3

#### THE BASIC EQUATIONS

1.2 The isentropic equation of state 4

1.3 The speed of sound 6

1.4 The continuity equation 7

1.5 The equation of motion 7

1.6 The vorticity equation 8

1.7 Irrotational motion 9

1.8 Conservation of circulation (Kelvin) 9

1.9 Velocity potential: pressure equation 9

1.10 Isentropic relations in steady flow. Critical values 10

1.11 The equation for the velocity potential 12

#### FORCES ON OBSTACLES

1.12 The momentum theorems 12

1.13 Forces acting on a rigid body 13

1.14 The jet-thrust paradox 14

1.15 D'Alembert's paradox: the force on semi-infinite bodies 16

1.16 The drag in a real fluid: inviscid flow models 17

1.17 The lift in a real fluid: Joukowski's hypothesis 22

#### BOUNDARY CONDITIONS

1.18 Solid boundaries 24

1.19 Design problems: given boundary pressures 25

1.20 Porous and perforated surfaces 26

1.21 Surfaces of separation: vortex sheets 28

1.22	Boundary conditions at the free surface of a heavy incompressible liquid	<i>page</i> 29
1.23	Boundary conditions at infinity	31
1.24	Miscellaneous boundary conditions	32
1.25	Existence and uniqueness theorems	33

## *Chapter 2.* GENERAL THEORY OF TWO-DIMENSIONAL FLOW

### INTRINSIC EQUATIONS: THE STREAM FUNCTION

2.1	Summary of basic equations	35
2.2	Intrinsic equations: natural coordinates	36
2.3	The stream function	37
2.4	Linear shear flow	38

### IRROTATIONAL STEADY FLOW

2.5	The equations for $q$ and $\theta$	39
2.6	Transformation to the $(\phi, \psi)$ -plane	39
2.7	The hodograph transformation	40
2.8	Non-linear boundary conditions	42

### LINEAR THEORIES

2.9	Linearization in the $(\phi, \psi)$ -plane	44
2.10	The tangency point	45
2.11	Linearization in the $(x, y)$ -plane	47
2.12	Comparison of the two theories	49
2.13	Relations for the velocity, pressure and density	49

### COMPLEX VARIABLE FORMULATION

2.14	Complex variable equations for the tangent gas	52
2.15	The relation between the $w$ - and $z$ -planes	53
2.16	Incompressible flow	54
2.17	Linear perturbation theory	55
2.18	The force and moment acting on a streamline	56
2.19	Blasius's formulae	58

## CONTENTS

vii

### PERTURBATIONS OF STEADY IRROTATIONAL FLOW

2.20	The possible perturbations	<i>page</i> 59
2.21	Unsteady irrotational flow	60
2.22	Small vorticity flow	63
2.23	Quasi-plane flows	64

## PART II. COMPLEX VARIABLE THEORY

### *Chapter* 3. CAUCHY INTEGRALS AND THEIR APPLICATION TO HARMONIC FUNCTION BOUNDARY VALUE PROBLEMS

#### CAUCHY'S THEOREM

3.1	Introduction	71
3.2	The residue theorem	73
3.3	The point at infinity	75

#### CAUCHY INTEGRALS

3.4	Cauchy's integral formula	76
3.5	Cauchy principal values	77
3.6	The Riemann–Stieltjes integral: the delta function	79
3.7	The Plemelj formulae	82
3.8	Derivatives of Cauchy integrals	83
3.9	Inversion formulae: the Poincaré–Bertrand formula	84

#### BOUNDARY SINGULARITIES

3.10	Singularities in the density function	86
3.11	Extension of the residue theorem	88
3.12	Boundary passing through the point at infinity	90

#### THE FUNDAMENTAL PROBLEMS OF POTENTIAL THEORY

3.13	The Dirichlet and Neumann problems	92
3.14	Solutions of the fundamental problems for a circle	
	(1) The continuation method	94
	(2) The image method	96
3.15	The 'Schwarzian inversion' or 'reflexion' principle	98

3.16	Solutions of the fundamental problems for an infinite strip	<i>page</i> 99
3.17	The two-sided character of the point at infinity	103
3.18	Basic solution for the upper half-plane	104

## GREEN'S FUNCTION METHODS

3.19	The complex variable form of Green's formulae	104
3.20	Green's function	107

*Chapter 4.* MIXED AND PERIODIC BOUNDARY CONDITIONS

4.1	The various types of mixed boundary conditions	110
-----	--	-----

## INFINITE AND SEMI-INFINITE STRIPS

4.2	Simply-mixed boundary conditions	112
4.3	Periodic boundary conditions for the upper half-plane	113

## THE THETA, ZETA AND JACOBIAN ELLIPTIC FUNCTIONS

4.4	The theta functions	116
4.5	The zeta functions	118
4.6	Jacobian elliptic functions	120
4.7	Some useful relations between the zeta and Jacobian functions	123
4.8	Jacobi's incomplete elliptic functions of the second and third kinds	125

## RECTANGLES

4.9	Periodic boundary conditions for the infinite strip	126
4.10	Alternative form for the solution	129
4.11	Limiting cases	130
4.12	Mixed and periodic boundary conditions for the infinite strip	131
4.13	The various boundary value problems for the rectangle	135
4.14	The Dirichlet and mixed problems for the rectangle	138

## THE RIEMANN-HILBERT PROBLEM

4.15	The boundary value problem	143
4.16	The reduction of the boundary value problem to two Dirichlet problems	144
4.17	General solution for a singly-connected region	145

## CONTENTS

ix

4.18	Examples	
	(1) The infinite strip	<i>page</i> 146
	(2) The upper half-plane	148
4.19	Mixed periodic boundary conditions	148
4.20	An example: the upper half-plane	151

### THE POINCARÉ PROBLEM

4.21	Reduction of the problem to an integral equation	152
4.22	The Poincaré problem in an infinite strip	154
4.23	Explicit solution of a special case by the use of Laplace transforms	155

## *Chapter 5.* CONFORMAL MAPPING

### BASIC MAPPING FORMULAE

5.1	General principles	157
5.2	The elementary transformations	160
5.3	The Schwarz–Christoffel mapping theorem	163
5.4	The basic integral equation of mapping theory	167
5.5	A first generalization of the mapping theorem	170
5.6	A second generalization of the mapping theorem	172
5.7	The mapping of nearly circular domains	175
5.8	The Theodorsen–Garrrick method for ‘star-shaped’ contours	176
5.9	The mapping of doubly-connected regions	178

### MAPPING AND FLUID MOTIONS

5.10	The relation between conformal mapping and incompressible fluid motion	180
5.11	The use of mapping methods in fluid motion problems	183

### SOME IMPORTANT CONFORMAL TRANSFORMATIONS

5.12	Flow in channels with mixed boundary conditions	184
5.13	Non-circulating flow about a cylinder	185
5.14	Circulating flow about a cylinder	187

5.15	Circulating flow about a cylinder in a stream of finite width	<i>page</i> 189
5.16	The use of simultaneous Schwarz–Christoffel transformations	192

## GREEN'S FUNCTION

5.17	The relation between Green's function and conformal mapping	195
------	---	-----

## PART III. APPLICATIONS

*Chapter 6. GENERAL ACCOUNT OF METHODS:  
SOURCES, DOUBLET AND VORTICES*

6.1	Recapitulation	201
-----	----------------	-----

## BOUNDARY CONDITIONS

6.2	Unmixed boundary conditions	204
6.3	Porous boundaries	205
6.4	Perforated boundaries	206
6.5	Mixed boundary conditions	208
6.6	Boundary conditions at the point at infinity	208
6.7	Circulation about and mass flow from an isolated body	209
6.8	Boundary conditions in unsteady flow	210

## SOURCES, SINKS AND VORTICES

6.9	Sources and sinks	212
6.10	Vortices	214
6.11	Source in a uniform stream	215
6.12	Doublet in a uniform stream	217
6.13	Vortex in a uniform stream	219

## GENERAL METHODS

6.14	Outline of the general procedure of solving fluid motion problems	221
------	---	-----

*Chapter 7. FLOW IN CHANNELS*

7.1	Introduction	<i>page</i> 224
FLOW THROUGH GIVEN CHANNELS		
7.2	General theory	226
7.3	The displacement function method	229
7.4	Rectilinear walls	230
7.5	A simple example: a step in a channel	232
7.6	A symmetrical cylinder in a channel	233
7.7	A numerical example: the circular cylinder	238
7.8	Wind-tunnel blockage	240
7.9	The increase in drag due to blockage	244
CHANNEL DESIGN		
7.10	General theory	245
7.11	Sectionally constant wall pressures	246
7.12	Design of a bend in a channel	247
SIMPLY-MIXED BOUNDARY CONDITIONS		
7.13	General theory	248
7.14	Flow of a jet, deflected around a given curved surface	250
7.15	An example of a channel design	253
7.16	The flow of a stream up a step, with boundary-layer separation	255
7.17	Helmholtz flow of a jet impinging normally on a flat plate	256
7.18	A generalization of Borda's mouthpiece	257
7.19	Flow from a nozzle	261
SYMMETRICAL JET FLOWS		
7.20	A symmetrical cylinder in a jet	264
7.21	Blockage in an open 'wind-tunnel'	266
7.22	The setting of 'streamline' walls	267
7.23	An example: the circular cylinder	269
DOUBLY-MIXED BOUNDARY CONDITIONS		
7.24	General theory	272
7.25	Flow over a step	273
7.26	Design of a channel bend	275

## POROUS AND PERFORATED WALLS

7.27	General theory	<i>page</i> 276
7.28	Blockage in a porous-walled tunnel	279
7.29	Perforated wind-tunnels	284

## SLOTTED WALLS

7.30	Mathematical formulation	287
7.31	The mean boundary conditions	290
7.32	Blockage in a slotted tunnel	294

## BRANCHED CHANNELS

7.33	Transformation of the $w$ -plane	297
7.34	Solution in the $\zeta$ -plane	299
7.35	Flow into a cascade of channels	300

*Chapter 8. AEROFOIL THEORY*

8.1	Introduction	301
-----	--------------	-----

## GENERAL AEROFOIL THEORY

8.2	The conformal transformations	302
8.3	The solution for a given aerofoil	304
8.4	The closure conditions	306
8.5	The profile centre and the circulation centroid	307
8.6	Conjugate equations for aerofoil design	310
8.7	The effect of a change of incidence	311
8.8	Incompressible flow	314
8.9	The displacement function	315

## FORCES ACTING ON AEROFOILS

8.10	The lift	316
8.11	The moment	317
8.12	The centre of pressure and the aerodynamic centre	319
8.13	Estimates for the thickness factor	321

## THIN AEROFOIL THEORY

8.14	The iterative solution of the basic integral equation	323
8.15	The principal equations of thin aerofoil theory	324
8.16	Round-nosed aerofoils	327



## CONTENTS

xiii

8.17	Aerofoils with finite trailing edge angles	<i>page</i> 329
8.18	The effect of round noses and trailing edge angles on thin aerofoil theory	332
8.19	Thin aerofoil design	334
8.20	Aerofoils with hinged flaps	335
8.21	Motion in a curved path	337

### THICK AEROFOIL THEORY

8.22	Theodorsen's theory for incompressible flow	338
8.23	Woods's method for subsonic flow	342
8.24	Other methods	343
8.25	Thick aerofoil design	345

### MIXED BOUNDARY CONDITIONS

8.26	Generalized aerofoil theory	346
8.27	An aerofoil of uniform porosity	350
8.28	A thin aerofoil with uniform suction pressure	352
8.29	Sources and sinks on aerofoils	355
8.30	Bubbles on thin aerofoils and aerofoils with spoilers	357
8.31	Aerofoil design	361
8.32	Finite cavities behind hydrofoils	362

## *Chapter 9.* UNSTEADY AEROFOIL THEORY

### MATHEMATICAL FORMULATION

9.1	Basic concepts and approximations	364
9.2	Solution of the boundary value problem	367

### NON-UNIFORM PERTURBATIONS

9.3	The strength of the vortex sheet	369
9.4	The pressure distribution on the aerofoil	373
9.5	The lift and moment	375

### HARMONIC OSCILLATIONS

9.6	A rigid aerofoil	375
9.7	Oscillations of long duration	378
9.8	Oscillations of hinged flaps	378

	GUSTS	
9.9	Sharp-edged upgusts	<i>page</i> 379
9.10	Graded gusts	381
	ACCELERATED MOTION	
9.11	General theory	381
9.12	Uniform acceleration	382
	COMPRESSIBLE FLOW	
9.13	Generalization of the theory	383
9.14	The strength of the vortex sheet	385
9.15	The pressure distribution on the aerofoil	387
	<i>Chapter 10. FLOW PAST SEMI-INFINITE PROFILES</i>	
10.1	Introduction	388
	SYMMETRICAL FLOWS	
10.2	General theory	390
10.3	Example: incompressible flow past a wedge	391
10.4	Linear perturbation theory	392
	ASYMMETRIC PROFILES	
10.5	The basic equations	393
10.6	The auxiliary conditions	394
10.7	The drag and lift forces	396
10.8	The moment about the leading edge	399
10.9	An example: a small nose-flap	400
10.10	Profiles of constant downstream width	402
10.11	Linear theory	404
	MIXED BOUNDARY CONDITIONS	
10.12	The general equations for a sting of given pressure distribution	405
10.13	Expressions for the pressure, lift and moment	406
	THE JET FLAP	
10.14	Description	407
10.15	The jet boundary condition	409
10.16	The leading edge potential	411

## CONTENTS

xv

10.17	A relation between the derivatives of the lift coefficient <i>page</i>	413
10.18	The lift and moment coefficients	414
10.19	Integro-differential equations for the jet slope	416
10.20	An approximation for the jet slope	419
10.21	Explicit expressions for the lift and moment coefficients and their derivatives	422

### *Chapter 11. WAKES AND CAVITIES*

#### GENERAL DESCRIPTION

11.1	Cavities, bubbles and wakes	426
11.2	The points of flow separation	428
11.3	Conditions in the wake	430
11.4	Physical reality	431

#### THEORY OF STEADY WAKE FLOW

11.5	The integral equation	433
11.6	Flow in the neighbourhood of a separation point	435
11.7	Flow in the neighbourhood of infinity	437
11.8	The lift and drag forces: (a) Aerodynamic wakes, (b) Cavitation wakes	441
11.9	Pressure distribution in aerodynamic wakes	443
11.10	Two simple examples of Helmholtz flow	444
11.11	Theory of aerofoil spoilers	447
11.12	Curved obstacles	449
11.13	An example: the circular cylinder	452

#### THEORY OF UNSTEADY WAKE FLOW

11.14	General description	454
11.15	The basic mathematical theory: fixed separation points	457
11.16	The wake conditions	459
11.17	The pressure distribution and forces on the obstacle	460
11.18	Non-uniform motion (Helmholtz flow)	464
11.19	Separation points at the same steady potential	467
11.20	Harmonic motions	469
11.21	Unsteady motion of a flat plate normal to the undisturbed stream	472
11.22	An oscillating aerofoil fitted with a spoiler	475

## FINITE CAVITIES

11.23	Riabouchinsky flow	page 477
11.24	Flow past a normal plate in a channel	480
11.25	Re-entrant jets	483

*Chapter 12. CASCADES OF AEROFOILS*

12.1	Some general relations	487
------	------------------------	-----

## STEADY DIRICHLET FLOW

12.2	The conformal transformations	489
12.3	General solution	494
12.4	The inlet and outlet conditions	495
12.5	The closure conditions	496
12.6	The direct and indirect problems of cascade theory	497
12.7	The force and moment acting on a cascade blade	498
12.8	An example: a cascade of flat plates in incompressible flow	500

## STEADY HELMHOLTZ FLOW

12.9	The conformal transformations	502
12.10	The boundary value problem and its solution	505
12.11	Inlet and outlet conditions	506
12.12	Example: a cascade of stalled flat plates	507
12.13	Incompressible flow through a grid of normal plates	509

## UNSTEADY FLOW

12.14	Description of problem	510
12.15	General theory	513
12.16	The strength of the vortex sheet	514
12.17	The pressure distribution	517
12.18	The lift and moment acting on a blade of the cascade	519
12.19	The generalized Theodorsen function	520

*Chapter 13. LIFTING AEROFOILS IN CHANNELS AND JETS*

## THE CONFORMAL TRANSFORMATIONS

13.1	An aerofoil in a general position in a stream of finite width	<i>page</i> 522
13.2	An aerofoil centrally placed in a stream of finite width	525
13.3	An aerofoil near a boundary in a stream of infinite width	526

## AEROFOILS IN CHANNELS

13.4	The general solution	527
13.5	Auxiliary conditions	528
13.6	Fourier series expansions	530
13.7	Design of a bend in a channel with a single turning vane	531
13.8	Exact theory for a flat plate in a straight channel	532
13.9	Approximate theory for a flat plate	535
13.10	Linear perturbation theory	538
13.11	Aerofoil in the centre of the channel	542
13.12	Ground effect on aerofoils	546

## AEROFOILS IN JETS

13.13	The general solution	546
13.14	The jet deflexion angle and the closure conditions	548
13.15	Linear perturbation theory	549
13.16	A flat plate in a jet of incompressible fluid	551
13.17	Aerofoil in the centre of a jet	553

## AEROFOILS BETWEEN POROUS WALLS

13.18	The mixed boundary value problem	554
13.19	The general solution	556
13.20	Some special cases	558
13.21	The lift and moment coefficients	560

*Chapter 14. UNSTEADY MOTION OF AEROFOILS  
IN CHANNELS AND JETS*UNSTEADY MOTION OF AN AEROFOIL  
IN A CLOSED WIND TUNNEL

14.1	The basic equations	563
14.2	The strength of the vortex sheet	566

14.3	The pressure distribution	<i>page</i> 569
14.4	The lift and moment	571
14.5	Harmonic motion of a rigid aerofoil	572

## UNSTEADY MOTION OF AN AEROFOIL IN A FREE JET

14.6	The basic equations	573
14.7	The strength of the vortex sheet	574
14.8	The pressure distribution	575
14.9	The lift and moment	576

<i>Concluding remarks</i>	577
---------------------------	-----

<i>References</i>	579
-------------------	-----

<i>Index</i>	589
--------------	-----