CONTENTS

Part A—Formulation of Variational Principles in Elasticity and Plasticity

INTRODUCTION 3

CHAPTER 1. SMALL DISPLACEMENT THEORY OF ELASTICITY IN RECTANGULAR CARTESIAN COORDINATES 11
1.1. Presentation of a Problem in Small Displacement Theory 11
1.2. Conditions of Compatibility 14
1.3. Stress Functions 16
1.4. Principle of Virtual Work 18
1.5. Approximate Method of Solution Based on the Principle of Virtual Work 19
1.6. Principle of Complementary Virtual Work 22
1.7. Approximate Method of Solution Based on the Principle of Complementary Virtual Work 24
1.8. Relations between Conditions of Compatibility and Stress Functions 28
1.9. Some Remarks 30
Exercises 33

CHAPTER 2. VARIATIONAL PRINCIPLES IN THE SMALL DISPLACEMENT THEORY OF ELASTICITY 42
2.1. Principle of Minimum Potential Energy 42
2.2. Principle of Minimum Complementary Energy 45
2.3. Generalization of the Principle of Minimum Potential Energy 48
2.4. Derived Variational Principles 50
2.5. Rayleigh–Ritz Method—(1) 55
2.6. Variation of the Boundary Conditions and Castigliano's Theorem 57
2.7. Free Vibrations of an Elastic Body 61
2.8. Rayleigh–Ritz Method—(2) 64
2.9. Some Remarks 67
Exercises 70

CHAPTER 3. FINITE DISPLACEMENT THEORY OF ELASTICITY IN RECTANGULAR CARTESIAN COORDINATES 79
3.1. Analysis of Strain 79
3.2. Analysis of Stress and Equations of Equilibrium 83
3.3. Transformation of the Stress Tensor 86
3.4. Stress–Strain Relations 87
3.5. Presentation of a Problem 88
3.6. Principle of Virtual Work 91
3.7. Strain Energy Function 93
3.8. Principle of Stationary Potential Energy 96
3.9. Generalization of the Principle of Stationary Potential Energy 97
3.10. Energy Criterion for Stability 99
3.11. The Euler Method for Stability Problem 101
3.12. Some Remarks 104
Exercises 105

CHAPTER 4. THEORY OF ELASTICITY IN CURVILINEAR COORDINATES 107
4.1. Geometry before Deformation 107
4.2. Analysis of Strain and Conditions of Compatibility 112
4.3. Analysis of Stress and Equations of Equilibrium 115
4.4. Transformation of the Strain and Stress Tensors 117
4.5. Stress–Strain Relations in Curvilinear Coordinates 120
4.6. Principle of Virtual Work 121
4.7. Principle of Stationary Potential Energy and its Generalizations 123
4.8. Some Specializations to Small Displacement Theory in Orthogonal Curvilinear Coordinates 125
Exercises 127

CHAPTER 5. EXTENSIONS OF THE PRINCIPLE OF VIRTUAL WORK AND RELATED VARIATIONAL PRINCIPLES 138
5.1. Initial Stress Problems 138
5.2. Stability Problems of a Body with Initial Stresses 142
5.3. Initial Strain Problems 144
5.4. Thermal Stress Problems 147
5.5. Dynamical Problems 150
5.6. Dynamical Problems of an Elastic Body with Initial Stresses 156
5.7. Dynamical Problems of an Unrestrained Elastic Body 161
Exercises 168

CHAPTER 6. TORSION OF BARS 173
6.1. Saint-Venant Theory of Torsion 173
6.2. The Principle of Minimum Potential Energy and its Transformation 177
6.3. Torsion of a Bar with a Hole 180
6.4. Torsion of a Bar with Initial Stresses 183
6.5. Upper and Lower Bounds of Torsional Rigidity 188
Exercises 195

CHAPTER 7. BEAMS 207
7.1. Elementary Theory of a Beam 207
7.2. Bending of a Beam 210
7.3. Principle of Minimum Potential Energy and its Transformation 214
7.4. Free Lateral Vibration of a Beam 216
7.5. Large Deflection of a Beam 220
7.6. Buckling of a Beam 222
7.7. Free Lateral Vibration of a Rotating Beam 226
7.8. A Beam Theory Including the Effect of Transverse Shear Deformation 229
7.9. Beam with Small Initial Curvature 233
7.10. Some Remarks 237
Exercises 239

CHAPTER 8. PLATES 253
8.1. Stretching and Bending of a Plate 253
8.2. A Problem of Stretching and Bending of a Plate 256
8.3. Principle of Minimum Potential Energy and its Transformation for the Stretching of a Plate 262
8.4. Principle of Minimum Potential Energy and its Transformation for the Bending of a Plate 265
8.5. Large Deflection of a Plate in Stretching and Bending 266
8.6. Buckling of a Plate 270
8.7. Thermal Stresses in a Plate 273
8.8. A Thin Plate Theory Including the Effect of Transverse Shear Deformation 276

8.9. Thin Shallow Shell 280
8.10. Some Remarks 286
Exercises 289

CHAPTER 9. SHELLS 306
9.1. Geometry before Deformation 306
9.2. Analysis of Strain 311
9.3. Analysis of Strain under the Kirchhoff–Love Hypothesis 315
9.4. A Linearized Thin Shell Theory under the Kirchhoff–Love Hypothesis 317
9.5. Simplified Formulations 322
9.6. A Simplified Linear Theory under the Kirchhoff–Love Hypothesis 324
9.7. A Nonlinear Thin Shell Theory under the Kirchhoff–Love Hypothesis 325
9.8. A Linearized Thin Shell Theory Including the Effect of Transverse Shear Deformations 327
9.9. Some Remarks 330
Exercises 333

CHAPTER 10. STRUCTURES 342
10.1 Finite Redundancy 342
10.2. Deformation Characteristics of a Truss Member and Presentation of a Truss Problem 343
10.3. Variational Formulations of the Truss Problem 346
10.4. The Force Method Applied to the Truss Problem 348
10.5. A Simple Example of a Truss Structure 351
10.6. Deformation Characteristics of a Frame Member 353
10.7. The Force Method Applied to a Frame Problem 355
10.8. Notes on the Force Method Applied to Semi-monocoque Structures 360
10.9. Notes on the Stiffness Matrix Method Applied to Semi-monocoque Structures 364
Exercises 369

CHAPTER 11. THE DEFORMATION THEORY OF PLASTICITY 376
11.1. The Deformation Theory of Plasticity 376
11.2. Strain-hardening Material 378
11.3. Perfectly Plastic Material 380
11.4. A Special Case of Hencky Material 383

CHAPTER 12. THE FLOW THEORY OF PLASTICITY 386
12.1. The Flow Theory of Plasticity 386
12.2. Strain-hardening Material 388
12.3. Perfectly Plastic Material 391
12.4. The Prandtl–Reuss Equation 393
12.5. The Saint-Venant–Levy–Mises Equations 395
12.6. Limit Analysis 399
12.7. Some Remarks 402

Part B—Variational Principles as a Basis of the Finite Element Method

INTRODUCTION 407

CHAPTER 13. CONVENTIONAL AND MODIFIED VARIATIONAL PRINCIPLES IN THE SMALL DISPLACEMENT THEORY OF ELASTOSTATICS 411
13.1. Presentation of a Problem 411
13.2. Conventional Variational Principles 413
13.3. Finite Elements 417

13.4. Modified Variational Principles 421
13.5. Modified Variational Principles—continued 427
Exercises 430

CHAPTER 14. Conventional and Modified Variational Principles in the Finite Displacement Theory of Elastostatics 432
14.1. Presentation of a Problem 432
14.2. Conventional Variational Principles 434
14.3. Derivation of Modified Variational Principles from the Principle of Stationary Potential Energy 436
14.4. A Formulation for the Compatible Model and the Modified Incremental Stiffness Method 440
14.5. Note on the Principle of Stationary Complementary Energy in Nonlinear Elasticity 442

CHAPTER 15. Conventional and Modified Variational Principles in Elastodynamics 446
15.1. Presentation of a Problem in the Small Displacement Theory of Elastodynamics 446
15.2. Conventional Variational Principles in the Small Displacement Theory of Elastodynamics 447
15.3. Gurtin's Principle 453

CHAPTER 16. Two Incremental Theories for a Solid-Body Problem with Geometrical and Material Nonlinearities 456
16.1. Introduction 456
16.2. Definitions of Strains 457
16.3. Definitions of Stresses 459
16.4. Relations between Stress and Strain Increments 466
16.5. Total Lagrangian Approach 467
16.6. Updated Lagrangian Approach 470
Exercises 473

CHAPTER 17. Variational Principles for the Bending of Elastic Plates 475
17.1. Conventional Variational Principles in the Small Displacement Theory under Kirchhoff Hypothesis 475
17.2. Modified Variational Principles in the Small Displacement Theory under Kirchhoff Hypothesis 484
17.3. Herrmann's Principle 489
17.4. Variational Principles for the Large Deflection of a Plate in Stretching and Bending under the Kirchhoff Hypothesis 490
17.5. Variational Principles for Marguerre's Theory of a Thin Shallow Shell 494
17.6. Conventional Variational Principles for the Bending of a Thin Plate including the Effect of Transverse Shear Deformation 497
17.7. Some Recent Variational Treatments of Plate Bending Problems* 499
Exercises 508

CHAPTER 18. Notes on Discrete Analysis 510
18.1. Introduction 510
18.2. Method of Weighted Residuals 511
18.3. Rayleigh–Ritz Method 515
18.4. Some Remarks on Discrete Analysis 517
18.5. Boundary Element Method 518
18.6. An Application of BEM to a Nonlinear Free Surface Fluid Problem 522
18.7. Some Remarks on the BEM 528
Exercises 539

APPENDIX A.	Notes on the Stationary Property of a Function	549
APPENDIX B.	Variational Principles in Dynamics of a Sytem of Particles	556
APPENDIX C.	Notes on the Principle of Vprhal Work	568
APPENDIX D.	Notes on the Strain Energy Function and Complementary Energy Function	572
APPENDIX E.	Notes on Several Kinds of Stress Tensors in Finite Displacement Theory	576
APPENDIX F.	Vectors and Tensors	584
APPENDIX G.	Coupling of Bending and Torsion of a Beam	587
APPENDIX H.	A Beam Theory including the Effect of Transverse Shear Deformation	595
APPENDIX I.	Stress–strain Relations for a Thin Plate	599
APPENDIX J.	A Theory of Plate Bending including the Effect of Transverse Shear Deformation	601
APPENDIX K.	Specializations to Several Kinds of Shells	604
APPENDIX L.	A Note on the Haar–Karman Principle	608
APPENDIX M.	Notes on Variational Principles for Quasi-static Problems and in the Theory of Creep	610
APPENDIX N.	Notes on the Boundary Element Method	616
INDEX		627