

CONTENTS

Preface xi

Brief guide on the use of symbols xiii

1.
INTRODUCTION 1

1.1

The nature of turbulence 1

Irregularity 1. Diffusivity 2. Large Reynolds numbers 2. Three-dimensional vorticity fluctuations 2. Dissipation 3. Continuum 3. Turbulent flows are flows 3.

1.2

Methods of analysis 4

Dimensional analysis 5. Asymptotic invariance 5. Local invariance 6.

1.3

The origin of turbulence 7

1.4

Diffusivity of turbulence 8

Diffusion in a problem with an imposed length scale 8. Eddy diffusivity 10. Diffusion in a problem with an imposed time scale 11.

1.5

Length scales in turbulent flows 14

Laminar boundary layers 14. Diffusive and convective length scales 15. Turbulent boundary layers 16. Laminar and turbulent friction 17. Small scales in turbulence 19. An inviscid estimate for the dissipation rate 20. Scale relations 21. Molecular and turbulent scales 23.

1.6

Outline of the material 24

2.

TURBULENT TRANSPORT OF MOMENTUM AND HEAT 27

2.1

The Reynolds equations 27

The Reynolds decomposition 28. Correlated variables 29. Equations for the mean flow 30. The Reynolds stress 32. Turbulent transport of heat 33.

2.2

Elements of the kinetic theory of gases 34

Pure shear flow 34. Molecular collisions 35. Characteristic times and lengths 38. The correlation between v_1 and v_2 38. Thermal diffusivity 39.

2.3

Estimates of the Reynolds stress 40

Reynolds stress and vortex stretching 40. The mixing-length model 42. The length-scale problem 44. A neglected transport term 45. The mixing length as an integral scale 45. The gradient-transport fallacy 47. Further estimates 49. Recapitulation 49.

2.4

Turbulent heat transfer 50

Reynolds' analogy 51. The mixing-length model 51.

2.5

Turbulent shear flow near a rigid wall 52

A flow with constant stress 54. Nonzero mass transfer 55. The mixing-length approach 55. The limitations of mixing-length theory 57.

3.

THE DYNAMICS OF TURBULENCE 59

3.1

Kinetic energy of the mean flow 59

Pure shear flow 60. The effects of viscosity 62.

3.2

Kinetic energy of the turbulence 63

Production equals dissipation 64. Taylor microscale 65. Scale relations 67. Spectral energy transfer 68. Further estimates 69. Wind-tunnel turbulence 70. Pure shear flow 74.

3.3

Vorticity dynamics 75

Vorticity vector and rotation tensor 76. Vortex terms in the equations of motion 76. Reynolds stress and vorticity 78. The vorticity equation 81. Vorticity in turbulent flows 84. Two-dimensional mean flow 85. The dynamics of $\Omega_j \Omega_j$ 86. The equation for $\overline{\omega_j \omega_j}$ 86. Turbulence is rotational 87. An approximate vorticity budget 88. Multiple length scales 92. Stretching of magnetic field lines 93.

3.4

The dynamics of temperature fluctuations 95

Microscales in the temperature field 95. Buoyant convection 97. Richardson numbers 98. Buoyancy time scale 99. Monin-Oboukhov length 100. Convection in the atmospheric boundary layer 100.

4.

BOUNDARY-FREE SHEAR FLOWS 104

4.1

Almost parallel, two-dimensional flows 104

Plane flows 104. The cross-stream momentum equation 106. The streamwise momentum equation 108. Turbulent wakes 109. Turbulent jets and mixing layers 110. The momentum integral 111. Momentum thickness 112.

4.2

Turbulent wakes 113

Self-preservation 113. The mean-velocity profile 115. Axisymmetric wakes 118. Scale relations 119. The turbulent energy budget 120.

4.3

The wake of a self-propelled body 124

Plane wakes 125. Axisymmetric wakes 127.

4.4

Turbulent jets and mixing layers 127

Mixing layers 128. Plane jets 129. The energy budget in a plane jet 131.

4.5

Comparative structure of wakes, jets, and mixing layers 133

4.6

Thermal plumes 135

Two-dimensional plumes 136. Self-preservation 141. The heat-flux integral 142. Further results 142.

5.

WALL-BOUNDED SHEAR FLOWS 146

5.1

The problem of multiple scales 146

Inertial sublayer 147. Velocity-defect law 147.

5.2

Turbulent flows in pipes and channels 149

Channel flow 149. The surface layer on a smooth wall 152. The core region 153. Inertial sublayer 153. Logarithmic friction law 156. Turbulent pipe flow 156. Experimental data on pipe flow 157. The viscous sub-layer 158. Experimental data on the law of the wall 161. Experimental data on the velocity-defect law 162. The flow of energy 163. Flow over rough surfaces 164.

5.3

Planetary boundary layers 166

The geostrophic wind 166. The Ekman layer 167. The velocity-defect law 167. The surface layer 168. The logarithmic wind profile 169. Ekman layers in the ocean 170.

5.4

The effects of a pressure gradient on the flow in surface layers 171

A second-order correction to pipe flow 174. The slope of the logarithmic velocity profile 175.

5.5

The downstream development of turbulent boundary layers 177

The potential flow 179. The pressure inside the boundary layer 181. The boundary-layer equation 182. Equilibrium flow 184. The flow in the wall layer 185. The law of the wall 185. The logarithmic friction law 186. The pressure-gradient parameter 186. Free-stream velocity distributions 188. Boundary layers in zero pressure gradient 190. Transport of scalar contaminants 194.

6.

THE STATISTICAL DESCRIPTION OF TURBULENCE 197

6.1

The probability density 197

6.2

Fourier transforms and characteristic functions 201

The effects of spikes and discontinuities 203. Parseval's relation 205.

6.3

Joint statistics and statistical independence 207

6.4

Correlation functions and spectra 210

The convergence of averages 211. Ergodicity 212. The Fourier transform of $\rho(\tau)$ 214.

6.5

The central limit theorem 216

The statistics of integrals 218. A generalization of the theorem 220. More statistics of integrals 220.

7.

TURBULENT TRANSPORT 223

7.1

Transport in stationary, homogeneous turbulence 223

Stationarity 223. Stationary, homogeneous turbulence without mean velocity 224. The probability density of the Lagrangian velocity 226. The Lagrangian integral scale 229. The diffusion equation 230.

7.2

Transport in shear flows 230

Uniform shear flow 230. Joint statistics 232. Longitudinal dispersion in channel flow 233. Bulk velocity measurements in pipes 235.

7.3

Dispersion of contaminants 235

The concentration distribution 235. The effects of molecular transport 237. The effect of pure, steady strain 238. Transport at large scales 241.

7.4

Turbulent transport in evolving flows 241

Thermal wake in grid turbulence 242. Self-preservation 243. Dispersion relative to the decaying turbulence 245. The Gaussian distribution 246. Dispersion in shear flows 246.

8.

SPECTRAL DYNAMICS 248

8.1

One- and three-dimensional spectra 248

Aliasing in one-dimensional spectra 248. The three-dimensional spectrum 250. The correlation tensor and its Fourier transform 250. Two

common one-dimensional spectra 251. Isotropic relations 253. Spectra of isotropic simple waves 254.

8.2

The energy cascade 256

Spectral energy transfer 258. A simple eddy 258. The energy cascade 260.

8.3

The spectrum of turbulence 262

The spectrum in the equilibrium range 262. The large-scale spectrum 264.

The inertial subrange 264.

8.4

The effects of production and dissipation 267

The effect of dissipation 269. The effect of production 271. Approximate spectra for large Reynolds numbers 272.

8.5

Time spectra 274

The inertial subrange 277. The Lagrangian integral time scale 277. An approximate Lagrangian spectrum 278.

8.6

Spectra of passive scalar contaminants 279

One- and three-dimensional spectra 280. The cascade in the temperature spectrum 281. Spectra in the equilibrium range 282. The inertial-diffusive subrange 283. The viscous-convective subrange 284. The viscous-diffusive subrange 285. Summary 286.

Bibliography and references 288

Index 295