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ERRATA

- p. 9, 2nd line of the table: read  $\frac{1}{2}e_{xy} = \frac{1}{2}e_{yx}$ , etc., instead of  $e_{xy} = e_{yx}$ , etc.
- p. 61, 3rd and 4th lines of table: read  $Z_x$  instead of  $Z$
- p. 99, 13th line: read gr instead of gr-wt
- p. 99, 15th line: read 1.293 instead of 1,293
- p. 113, Eq. (1): read  $\nabla^2$  instead of  $\Delta$
- p. 181, Fig. 39b is upside down
- p. 233, last equation: read  $\tilde{z}_k = \tilde{x}_k + i\tilde{y}_k$  instead of  $\tilde{z}_k = \tilde{x}_k = i\tilde{y}_k$
- p. 234, 6th line: read  $z_k$  instead of  $\tilde{z}_k$
- p. 234, 7th line: read  $\tilde{z}_0$  instead of  $\tilde{z}$
- p. 289, 2nd line of footnotes: read p. 9 instead of p. 000
- p. 338, 22nd line: omit "and note that they are not equal"
- p. 347, last term of 14th line: read  $\frac{\partial A_\varphi}{\partial \varphi}$  instead of  $\frac{\partial A_\varphi}{\partial \varphi}$
- p. 348, third formula: read  $\nabla^2 A$  instead of  $(A \text{ grad})A$  and add:
 
$$(A \text{ grad})A = \begin{cases} A \cdot \text{grad } A_r - \frac{1}{r}(A_\delta^2 + A_\varphi^2) \\ A \cdot \text{grad } A_\delta + \frac{1}{r}(A_r A_\delta - A_\varphi^2 \cot \delta) \\ A \cdot \text{grad } A_\varphi + \frac{1}{r}(A_r A_\varphi + A_\delta A_\varphi \cot \delta) \end{cases}$$
- p. 354, line 5: read  $|M_e| = |M_i| = 4\pi\mu U r_e r_i^2 / (r_e^2 - r_i^2)$  instead of the values given for  $M_e$  and  $M_i$ , and replace the first part of the next sentence by "The difference  $-|M_e|/2\pi r_e + |M_i|/2\pi r_i$  equals  $2\mu U / (1 + r_e/r_i)$ , which, in the limit, becomes  $\mu U$ ;"
- p. 386, Eq. (1): read  $p_{ik}$  instead of  $i_k$