CONTENTS

PREFACE v

CONTENTS VII

CHAPTER I

VISCOELASTIC WAVES

by S. C. HUNTER

1. INTRODUCTION 3

2. THE LINEAR VISCOELASTIC SOLID 4
   2.1 Physical behaviour of anelastic solids 4
   2.2 Representation by Laplace transforms 8
   2.3 Representation by Fourier transform 11
   2.4 Representation by an operational modulus 15

3. THEORY OF THE PROPAGATION OF UNIAXIAL STRESS PULSES 16
   3.1 Statement of the problem 16
   3.2 Solution by Laplace transform 17
   3.3 Solution by Fourier transform 22
   3.4 Geometric dispersion 28

4. EXPERIMENTAL INVESTIGATIONS ON PULSE PROPAGATION 32

5. THE GENERAL EQUATIONS OF AN ISOTROPIC VISCOELASTIC SOLID . . . 44
   5.1 The generalised equations of viscoelasticity 44
   5.2 Application of integral transforms 48
   5.3 Quasi-static viscoelasticity 50
   5.4 Waves in three dimensions 52

REFERENCES 56

CHAPTER II

MATRICES OF TRANSMISSION IN BEAM PROBLEMS

by K. MARGUERRE

1. INTRODUCTION 61

2. THE VIBRATING BEAM 62

3. MATRICES FOR POINTS OF DISCONTINUITY: SUPPORTS, SINGLE MASSES, ETC. 65

4. RIGID SUPPORTS, Δ-MATRIX 66

5. Δ-MATRICES (CONTINUED) 73

6. "STIFF" SUPPORTS 75

7. APPLICATION OF THE MATRIX METHOD TO MORE COMPLEX PROBLEMS . 77

REFERENCES 82

CHAPTER III

DYNAMIC EXPANSION OF SPHERICAL CAVITIES IN METALS

by H. G. HOPKINS

1. INTRODUCTION 85

2. METHODS OF APPROACH AND OBJECTIVES 91

3. HOPKINSON'S SIZE-SCALING LAW 93

4. STRONG DISCONTINUITIES IN PHYSICAL QUANTITIES 93

5. ELASTIC DEFORMATIONS 98

6. SMALL ELASTIC-PLASTIC DEFORMATIONS 111
6.1 Perfectly-plastic material 113
6.2 Work-hardening material 128
6.3 Characteristics theory of equations 132

7. LARGE ELASTIC-PLASTIC DEFORMATIONS 135
7.1 Fundamental equations 135
7.2 Quasi-static motion 138
7.3 Dynamic motion 144
7.4 Characteristics theory of equations 158

8. CONCLUDING REMARKS 161

NOTATION 162

REFERENCES 163

CHAPTER IV

GENERAL THEOREMS FOR ELASTIC-PLASTIC SOLIDS

by W. T. KOITER

1. INTRODUCTION 167

2. BASIC ASSUMPTIONS AND STRESS-STRAIN RELATIONS 169
2.1 Small deformations 169
2.2 General stress-strain relations 171
2.3 Perfectly plastic materials 172
2.4 Work-hardening materials 174
2.5 Inversion of stress-strain relations 175
2.6 Historical remarks 178

3. UNIQUENESS THEOREMS
3.1 Boundary value problem for stress rates 181
3.2 Proof of uniqueness of stress rates 182
3.3 Uniqueness of strain rates 182
3.4 Uniqueness of stresses 183
3.5 Historical remarks 185

4. MINIMUM PRINCIPLES 186
4.1 Definitions 186
4.2 Formulation of principles for stress and strain rates 186
4.3 Proof of principles of par. 4.2 188

4.4 Principles for stresses and strains 189
4.5 Historical remarks 192
4.6 The Haar-Kármán principle 193

5. PLASTIC COLLAPSE THEOREMS AND LIMIT ANALYSIS 194
5.1 Definitions 194
5.2 Formulation of theorems 195
5.3 Proof of theorems 197
5.4 Uniqueness of stresses at collapse 198
5.5 Bounds on overall deformation 200
5.6 Historical remarks 201

6. SHAKEDOWN THEOREMS 203
6.1 Definitions 203
6.2 Formulation of theorems 206
6.3 Proof of Melan's theorem 207
6.4 Proof of first part of second theorem 208
6.5 Proof of second part of second theorem 209
6.6 Historical remarks 213

7. EXISTENCE OF SOLUTIONS 214
7.1 Dirichlet's principle 214
7.2 Basic inequality 214
7.3 Existence proof 215
7.4 Concluding remarks 217

REFERENCES 218

CHAPTER V

DISPERSION RELATIONS FOR ELASTIC WAVES IN BARS

by W. A. GREEN

1. INTRODUCTION 225

2. EXACT SOLUTIONS (POCHHAMMER AND CHREE) 228

3. ELEMENTARY APPROXIMATE THEORY 230

4. APPROXIMATE METHODS I 231
4.1 Rayleigh's method 231
4.2 Love's method 232
4.3 Timoshenko's method 232
4.4 Prescott's method 234
4.5 Method of Mindlin and Herrmann 235
4.6 Volterra's method 236
4.7 Bishop's correction 237
4.8 Kynch's method 238

5. APPROXIMATE METHODS II 239
5.1 Chree's method 240
5.2 Morse's method 240
5.3 Method of Kynch and Green 241

6. DISCUSSION 243

7. HIGH FREQUENCY SOLUTIONS FOR A CIRCULAR CYLINDER 250
7.1 Fundamental dispersion curves for complex modes 251
7.2 Dispersion curves for higher harmonics 254

NOTATION 260

REFERENCES 261

CHAPTER VI

THERMOELASTICITY. THE DYNAMICAL THEORY

by P. CHADWICK

1. INTRODUCTION 265

2. THE THERMOELASTIC EQUATIONS 269
2.1 The irreversible thermodynamics of an elastic solid 269
2.2 Non-equilibrium thermodynamic functions 272
2.3 Basic properties of the thermoelastic equations 276

3. PLANE HARMONIC THERMOELASTIC WAVES 280
3.1 Uncoupled waves 280
3.2 Coupled waves 282
3.3 Discussion and numerical results 288

4. BOUNDARY VALUE PROBLEMS. GENERAL CONSIDERATIONS 298
4.1 Classification of thermoelastic boundary value problems 299
4.2 Approximate forms of the thermoelastic equations 303

5. THERMOELASTIC BOUNDARY VALUE PROBLEMS 306
5.1 Thermoelastic Rayleigh waves 307
5.2 Thermoelastic vibrations of a circular cylinder 312
5.3 Radially symmetrical thermoelastic disturbances 316
5.4 Axially symmetrical thermoelastic disturbances 324

REFERENCES 328

CHAPTER VII

CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS

by B. A. BILBY

1. INTRODUCTION 331

2. BURGERS VECTOR AND TORSION TENSOR 335
2.1 Discrete dislocations 335
2.2 The continuously dislocated crystal 337
2.3 The geometry of the continuously dislocated crystal 339
2.4 Compatibility and the nature state 345

3. SHAPE, LATTICE AND DISLOCATION DEFORMATIONS 350
3.1 Types of deformation 350
3.2 Analytical relations 354
3.3 The dislocation motion tensor 358

4. ZERO LATTICE PURE STRAIN 359
4.1 General theory 359
4.2 Applications 365

5. DISLOCATION DENSITY AND STRESS 369
5.1 The incompatibility tensor 369
5.2 Surface dislocations 375
5.3 The determination of the deformations when the dislocation density is given 379
5.4 The determination of internal stress 379
5.5 Discrete dislocations 382
5.6 Discussion of plastic problems 383

6. GENERALISED SPACES 385
6.1 The natural state 385
6.2 Use of a Riemannian connexion 386
6.3 Non-Riemannian connexions 389
6.4 Discussion and generalisations 390
6.5 Application to non-linear elasticity 391

7. CONCLUSION 394

REFERENCES 396

CHAPTER VIII

ASYMMETRIC PROBLEMS OF THE THEORY OF ELASTICITY FOR A SEMI-INFINITE SOLID AND A THICK PLATE

by R. MUKI

1. INTRODUCTION 401

2. SOLUTION OF THE EQUATIONS OF EQUILIBRIUM BY HANKEL TRANSFORMS 401

3. SOLUTION OF THE THERMO-ELASTIC EQUATIONS BY HANKEL TRANSFORMS 407

4. STRESSES IN A SEMI-INFINITE ELASTIC SOLID UNDER THE COMPRESSIVE ACTION OF A RIGID BODY 410
4.1 Introduction 410
4.2 General solution 411
4.3 Indentation by a slightly inclined flat-ended cylinder 413

5. STRESSES IN A SEMI-INFINITE ELASTIC SOLID WITH A TANGENTIAL LOAD ON ITS SURFACE 419
5.1 Introduction 419
5.2 General solution 419
5.3 Example. 421

6. THERMAL STRESSES IN A SEMI-INFINITE ELASTIC SOLID AND A THICK PLATE UNDER STEADY DISTRIBUTION OF TEMPERATURE 423
6.1 Introduction 423
6.2 General solution for a semi-infinite solid 424
6.3 General solution for a thick plate 426
6.4 Stresses in a plate with a given surface distribution of temperature 429
6.5 Stresses in a plate with surface radiation 435

REFERENCES 439

AUTHOR INDEX 441

SUBJECT INDEX 445