Contents

Introduction 1

1 Experiments and Simple Models 7

1.1 Experimental Detection of Deterministic Chaos 7
1.2 The Periodically Kicked Rotator 12
Logistic Map 13
Hénon Map 13
Chirikov Map 14

2 Piecewise Linear Maps and Deterministic Chaos 15

2.1 The Bernoulli Shift 15
2.2 Characterization of Chaotic Motion 18
Liapunov Exponent 18
Invariant Measure 22
Correlation Function 24
2.3 Deterministic Diffusion 26

3 Universal Behavior of Quadratic Maps 31

3.1 Parameter Dependence of the Iterates 33
3.2 Pitchfork Bifurcations and the Doubling Transformation 35
Pitchfork Bifurcations 36
Supercycles 38
Doubling Transformation and α 40
Linearized Doubling Transformation and δ 41
3.3 Self-Similarity, Universal Power Spectrum and the Influence of External Noise 46
Self-Similarity in the Positions of the Cycle Elements 46
Hausdorff Dimension 48
Power Spectrum 50
Influence of External Noise 53
Behavior of the Logistic Map for $r_\infty < r$ 56

3.4 Parallels between Period Doubling and Phase Transitions 59
3.5 Experimental Support for the Bifurcation Route 62

4 The Intermittency Route to Chaos 67

4.1 Mechanisms for Intermittency 67
Type-I Intermittency 68
Length of the Laminar Region 71
4.2 Renormalization-Group Treatment of Intermittency 74
4.3 Intermittency and $1/f$-Noise 79
4.4 Experimental Observation of the Intermittency Route 86
Distribution of Laminar Lengths 87
Type-I Intermittency 88
Type-III Intermittency 89

5 Strange Attractors in Dissipative Dynamical Systems 91

5.1 Introduction and Definition of Strange Attractors 91
Baker's Transformation 95
Dissipative Hénon Map 96
5.2 The Kolmogorov Entropy 98
Definition of K 99
Connection of K to the Liapunov Exponents 101
Average Time over which the State of a Chaotic System Can Be Predicted 102
5.3 Characterization of the Attractor by a Measured Signal 103
Dimensions of a Strange Attractor 104
Correlation Integral 105
Reconstruction of the Attractor from a Time Series 106
Embedding Dimension 108
Seperation of Deterministic Chaos and External White Noise 109
Lower Bound for the Kolmogorov Entropy 109
Kaplan-Yorke Conjecture 111
5.4 Strange Attractors and the Onset of Turbulence 112
Hopf Bifurcation 112
Landau's Route to Turbulence 114
Ruelle-Takens-Newhouse Route to Chaos 114
Bénard Instability 115
Taylor Instability 117
5.5 Universal Properties of the Transition from Quasiperiodicity to Chaos 118
5.6 Routes to Chaos 126
Possibility of Three-Frequency Quasiperiodic Orbits 127

Frequency Locking 130
Crises 131
5.7 Pictures of Strange Attractors and Fractal Boundaries 132

6 Regular and Irregular Motion in Conservative Systems 137

6.1 Coexistence of Regular and Irregular Motion 139
Integrable Systems 139
Pertubation Theory and Vanishing Denominators 141
Stable Tori and KAM Theorem 143
Unstable Tori and Poincaré-Birkhoff Theorem 144
Homoclinic Points and Chaos 147
Arnold Diffusion 148
Examples of Classical Chaos 149
6.3 Strongly Irregular Motion and Ergodicity 151
Cat Map 152
Hierarchy of Classical Chaos 154
Three Classical K Systems 158

7 Chaos in Quantum Systems? 161

7.1 The Quantum Cat Map 163
7.2 A Quantum Particle in a Stadium 164
7.3 The Kicked Quantum Rotator 166

Outlook 173

Appendix 175

A Derivation of the Lorenz Model 175
B Stability Analysis and the Onset of Convection and Turbulence in the Lorenz Model 177
C The Schwarzian Derivative 179
D Renormalization of the One-Dimensional Ising Model 180
E Decimation and Path Integrals for External Noise 183
F Shannon's Measure of Information 187
Information Capacity of a Store 187
Information Gain 188
G Period Doubling for the Conservative Hénon Map 190

Remarks and References 197

Subject Index 217