

TABLE OF CONTENTS

	page
Chapter I. Introduction	1
A. The Realm of Computational Fluid Dynamics	1
B. Historical Outline of Computational Fluid Dynamics	2
C. Existence and Uniqueness of Solutions	6
D. Preliminary Remarks on Consistency, Convergence, and Stability of Solutions ...	7
Chapter II. Incompressible Flow Equations in Rectangular Coordinates	9
A. Primitive Equations	9
B. Stream Function and Vorticity Equations for Planar Flows	9
C. Conservation Form	11
D. Normalizing Systems	11
E. One-Dimensional Model Transport Equations	12
Chapter III. Basic Computational Methods for Incompressible Flow	15
A. Methods for Solving the Vorticity Transport Equation	18
1. some basic finite difference forms	18
a. Taylor series expansions	18
b. polynomial fitting	22
c. integral method	23
2. control volume approach	25
3. the conservative property	28
4. a description of instability	33
5. stability analyses	36
a. discrete perturbation stability analysis	36
b. von Neumann stability analysis	42
c. Hirt's stability analysis	46
d. summary and evaluation of stability criteria	48
e. the von Neumann analysis in multi-dimensional problems	51
6. one-step explicit methods; the midpoint leapfrog method	53
7. the DuFort-Frankel leapfrog method	61
8. the first upwind differencing method; artificial viscosity errors	64
9. the transportive property	67
10. transportive and conservative differencing	70
11. the second upwind differencing method	73
12. Adams-Bashforth and Crocco methods	74
13. Leith's method; phase errors, aliasing errors, and time-splitting	75
14. implicit methods	83
15. multi-step explicit methods	87
16. ADI methods	91
17. ADE methods	95
18. hopscotch method	99
19. the fourth-order methods of Roberts and Weiss and of Crowley	101
20. Fromm's method of zero average phase error	103
21. Arakawa's method	105
22. remarks on steady-flow methods	106
23. remarks on evaluating methods; behavioral errors	109
B. Methods for Solving the Stream Function Equation	113
1. direct methods	113
2. Richardson's method and Liebman's method	114
3. Southwell's residual relaxation method	117
4. successive over-relaxation (SOR) method	117
5. tactics and strategy	119
6. ADI methods	122
7. other iterative methods	123
8. EVP method	124
9. Fourier-series methods	131
10. higher order approximations	134
11. remarks on evaluating methods	137
C. Boundary Conditions for the Vorticity and Stream Function Equations	139
1. remarks on the dominant importance of computational boundary conditions ..	139
2. walls in the first mesh system	140
3. walls in alternate mesh systems	146
4. symmetry boundaries	148
5. upper boundary	149

(continued)

	page
6. upstream boundary	152
7. outflow boundary	154
8. "wiggles"	161
9. the downstream paradox	165
10. computational versus analytical boundary conditions	167
11. conditions at "infinity"	167
12. the sharp corners	168
a. boundary conditions at the sharp convex corner	169
b. convergence and accuracy at the sharp convex corner	171
D. Convergence Criteria and Initial Conditions	174
E. Pressure Solution	180
1. numerical cubature	180
2. Poisson equation for pressure	180
3. boundary conditions of the second kind on pressure	182
4. iterative solution methods	183
5. pressure level	184
F. Temperature Solution and Concentration Solutions	186
1. basic equations	186
2. retention of dissipation	187
3. finite-difference representation of dissipation	188
4. boundary conditions for temperature and concentration	189
5. source terms and stiff equations	191
G. Methods for Solving the Primitive Equations	194
1. general considerations	194
2. basic equations	194
3. boundary conditions in primitive variables	195
4. the MAC method	196
5. other methods using primitive variables	201
6. relative merits of the (ψ, ζ) and (u, v, P) systems	202
H. Three-Dimensional Flows	204
Chapter IV. Compressible Flow Equations in Rectangular Coordinates	209
A. Fundamental Difficulties	210
B. Customary Equations	210
C. Conservation Form	211
D. Supplemental Relations	214
E. Normalized Conservation Equations	216
F. Short-Form Equations	220
G. Existence of Shocks - Physical and Mathematical	222
Chapter V. Basic Computational Methods for Compressible Flow	225
A. Preliminary Considerations	226
1. shock-free methods and shock-patching methods	226
2. stability considerations	228
3. implicit methods	230
B. Methods for the Numerical Treatment of Shocks	230
C. Shock Smearing by Artificial Dissipation	232
D. Methods Using Explicit Artificial Viscosities	232
1. von Neumann-Richtmyer method	232
2. Landshoff's method and Longley's method	234
3. Rusanov's method	235
4. errors arising from artificial viscosities	237
E. Methods Using Implicit Artificial Damping	237
1. upwind differencing	237
2. the domain of influence and truncation error	239
3. PIC and FLIC	240
4. Lax's method	242
5. Lax-Wendroff method	244
6. two-step Lax-Wendroff methods	250
7. the method of Abarbanel and Zwas	254
8. other methods; flux-corrected transport algorithm of Boris	255
F. Viscous Terms in the Compressible Flow Equations	256
1. spatial difference forms	256
2. general considerations	257
3. methods for the viscous terms	257

(continued)

	page
G. Boundary Conditions for Compressible Flow	261
1. slip walls	261
a. slip walls in the first mesh system	262
b. slip walls in the second mesh system	264
2. no-slip walls	266
a. no-slip walls in the first mesh system	266
b. no-slip walls in the second mesh system	269
c. staggered mesh evaluation of density	272
3. sharp corners	275
4. symmetry surfaces	278
5. upstream boundary	279
6. downstream boundary	279
7. upper boundary	282
H. Convergence Criteria and Initial Conditions	284
I. Remarks on Subsonic and Supersonic Solutions	285
J. Higher Order Systems	286
Chapter VI. Other Mesh Systems, Coordinate Systems, and Equation Systems	287
A. Special Mesh Systems	288
B. Coordinate Transformations	292
C. Other Orthogonal Coordinate Systems	301
D. Other Systems of Equations	303
E. Areas of Future Development	312
Chapter VII. Recommendations of Programming, Testing, and Information Processing .	315
A. Computer Programming	316
B. Debugging and Testing	321
C. Information Processing	328
1. numbers	328
2. plots and motion pictures	331
3. diagnostic functionals	342
D. Closure	343
Appendix A. Tridiagonal Algorithm	345
Appendix B. On Artificial Viscosity	350
Problems	367
References and Bibliography	375