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¹ Appears now for the first time.

² Appeared in the First Edition.

ERRATUM.

Vol. I. p. 407, footnote. Add reference to Chree, *Camb. Phil. Trans.*, Vol. XIV. p. 250, 1887.