



# CONTENTS

## I BANACH SPACES

1. On structure, 3
2. The axioms, 4
3. Linear functionals, 10
4. The canonical map, 15
5. Subspaces and orthogonality, 16
6. The Hahn–Banach theorem, 18
7. Other topologies, 23
8. Examples and exercises, 28

## II LINEAR TRANSFORMATIONS

1. Preliminaries, 32
2. The adjoint transformation, 36
3. The boundedness of the inverse transformation, 37
4. Closed transformations, 42
5. The uniform boundedness principle, 45
6. Projections, 46
7. Topologies for transformations, 51
8. On range and null-space, 52
9. The mean-ergodic theorem, 54

## III HILBERT SPACE

1. Definition, 57
2. Linear functionals, 62
3. Orthonormal sets, 65
4. Unbounded transformations and their adjoints, 69
5. Projections, 72
6. Resolutions of the identity, 74

7. Unitary transformations, 79
8. Examples and exercises, 81

#### **IV SPECTRAL THEORY OF LINEAR TRANSFORMATIONS**

1. The setting, 86
2. The spectrum, 89
3. Integration procedures, 91
4. The fundamental projections, 92
5. A special case, 99
6. The spectral radius, 101
7. Analytic functions of operators, 102

#### **V THE STRUCTURE OF SELF-ADJOINT TRANSFORMATIONS**

1. Preliminary discussion, 106
2. Positive operators, 107
3. The point spectrum, 110
4. The partition into pure types, 112
5. The continuous spectrum, 115

#### **VI COMMUTATIVE BANACH ALGEBRAS**

1. Introduction, 122
2. Definitions and examples, 123
3. The regular representation, 125
4. Reducibility and idempotents, 127
5. Algebras which are fields, 128
6. Ideals, 130
7. Quotient algebras, 132
8. Homomorphisms and maximal ideals, 136
9. The radical, 141
10. The representation theory, 143
11. Illustrative examples and applications, 145

Selected References, 153

Index, 155

