

Contents

PREFACE

I. Introduction to Manifolds

1. Preliminary Comments on \mathbf{R}^n 1
2. \mathbf{R}^n and Euclidean Space 4
3. Topological Manifolds 6
4. Further Examples of Manifolds. Cutting and Pasting 11
5. Abstract Manifolds. Some Examples 14
- Notes 18

II. Functions of Several Variables and Mappings

1. Differentiability for Functions of Several Variables 20
2. Differentiability of Mappings and Jacobians 25
3. The Space of Tangent Vectors at a Point of \mathbf{R}^n 29
4. Another Definition of $T_a(\mathbf{R}^n)$ 32
5. Vector Fields on Open Subsets of \mathbf{R}^n 37
6. The Inverse Function Theorem 41
7. The Rank of a Mapping 46
- Notes 50

III. Differentiable Manifolds and Submanifolds

1. The Definition of a Differentiable Manifold 52
2. Further Examples 60
3. Differentiable Functions and Mappings 65
4. Rank of a Mapping. Immersions 69
5. Submanifolds 75
6. Lie Groups 81
7. The Action of a Lie Group on a Manifold. Transformation Groups 89
8. The Action of a Discrete Group on a Manifold 95
9. Covering Manifolds 100
- Notes 104

IV. Vector Fields on a Manifold

1. The Tangent Space at a Point of a Manifold 106
2. Vector Fields 115
3. One-Parameter and Local One-Parameter Groups Acting on a Manifold 122
4. The Existence Theorem for Ordinary Differential Equations 130
5. Some Examples of One-Parameter Groups Acting on a Manifold 138
6. One-Parameter Subgroups of Lie Groups 145
7. The Lie Algebra of Vector Fields on a Manifold 149
8. Frobenius' Theorem 156
9. Homogeneous Spaces 164
- Notes 171
- Appendix Partial Proof of Theorem 4.1 172

V. Tensors and Tensor Fields on Manifolds

1. Tangent Covectors 175
 - Covectors on Manifolds 176
 - Covector Fields and Mappings 178
2. Bilinear Forms. The Riemannian Metric 181
3. Riemannian Manifolds as Metric Spaces 185
4. Partitions of Unity 191
 - Some Applications of the Partition of Unity 193
5. Tensor Fields 197
 - Tensors on a Vector Space 197
 - Tensor Fields 199
 - Mappings and Covariant Tensors 200
 - The Symmetrizing and Alternating Transformations 201
6. Multiplication of Tensors 204
 - Multiplication of Tensors on a Vector Space 205
 - Multiplication of Tensor Fields 206
 - Exterior Multiplication of Alternating Tensors 207
 - The Exterior Algebra on Manifolds 211
7. Orientation of Manifolds and the Volume Element 213
8. Exterior Differentiation 217
 - An Application to Frobenius' Theorem 221
- Notes 225

VI. Integration on Manifolds

1. Integration in \mathbb{R}^n . Domains of Integration 227
 - Basic Properties of the Riemann Integral 228
2. A Generalization to Manifolds 233
 - Integration on Riemannian Manifolds 237
3. Integration on Lie Groups 241
4. Manifolds with Boundary 248
5. Stokes's Theorem for Manifolds with Boundary 256
6. Homotopy of Mappings. The Fundamental Group 263
 - Homotopy of Paths and Loops. The Fundamental Group 265
7. Some Applications of Differential Forms. The de Rham Groups 271
 - The Homotopy Operator 274

8.	Some Further Applications of de Rham Groups	278
	The de Rham Groups of Lie Groups	282
9.	Covering Spaces and the Fundamental Group	286
	Notes	292

VII. Differentiation on Riemannian Manifolds

1.	Differentiation of Vector Fields along Curves in R^n	294
	The Geometry of Space Curves	297
	Curvature of Plane Curves	301
2.	Differentiation of Vector Fields on Submanifolds of R^n	303
	Formulas for Covariant Derivatives	308
	$\nabla_{X_p} Y$ and Differentiation of Vector Fields	310
3.	Differentiation on Riemannian Manifolds	313
	Constant Vector Fields and Parallel Displacement	319
4.	Addenda to the Theory of Differentiation on a Manifold	321
	The Curvature Tensor	321
	The Riemannian Connection and Exterior Differential Forms	324
5.	Geodesic Curves on Riemannian Manifolds	326
6.	The Tangent Bundle and Exponential Mapping. Normal Coordinates	331
7.	Some Further Properties of Geodesics	338
8.	Symmetric Riemannian Manifolds	347
9.	Some Examples	353
	Notes	360

VIII. Curvature

1.	The Geometry of Surfaces in E^3	362
	The Principal Curvatures at a Point of a Surface	366
2.	The Gaussian and Mean Curvatures of a Surface	370
	The Theorema Egregium of Gauss	373
3.	Basic Properties of the Riemann Curvature Tensor	378
4.	The Curvature Forms and the Equations of Structure	385
5.	Differentiation of Covariant Tensor Fields	391
6.	Manifolds of Constant Curvature	399
	Spaces of Positive Curvature	402
	Spaces of Zero Curvature	404
	Spaces of Constant Negative Curvature	405
	Notes	410

REFERENCES	413
------------	-----

INDEX	417
-------	-----