

CONTENTS

Preface

vii

Chapter 1 Differentiable and Analytic Manifolds

1

1.1	Differentiable Manifolds	1
1.2	Analytic Manifolds	20
1.3	The Frobenius Theorem	25
1.4	Appendix	31
	Exercises	35

Chapter 2 Lie Groups and Lie Algebras

41

2.1	Definition and Examples of Lie Groups	41
2.2	Lie Algebras	46
2.3	The Lie Algebra of a Lie Group	51
2.4	The Enveloping Algebra of a Lie Group	55
2.5	Subgroups and Subalgebras	57
2.6	Locally isomorphic Groups	61
2.7	Homomorphisms	67
2.8	The Fundamental Theorem of Lie	72
2.9	Closed Lie Subgroups and Homogeneous Spaces. Orbits and Spaces of Orbits	74
2.10	The Exponential Map	84
2.11	The Uniqueness of the Real Analytic Structure of a Real Lie Group	92
2.12	Taylor Series Expansions on a Lie Group	94
2.13	The Adjoint Representations of \mathfrak{g} and G	101
2.14	The Differential of the Exponential Map	107

2.15	The Baker-Campbell-Hausdorff Formula	114
2.16	Lie's Theory of Transformation Groups	121
	Exercises	133

Chapter 3 Structure Theory

149

3.1	Review of Linear Algebra	149
3.2	The Universal Enveloping Algebra of a Lie Algebra	166
3.3	The Universal Enveloping Algebra as a Filtered Algebra	176
3.4	The Enveloping Algebra of a Lie Group	184
3.5	Nilpotent Lie Algebras	189
3.6	Nilpotent Analytic Groups	195
3.7	Solvable Lie Algebras	200
3.8	The Radical and the Nil Radical	204
3.9	Cartan's Criteria for Solvability and Semisimplicity	207
3.10	Semisimple Lie Algebras	213
3.11	The Casimir Element	216
3.12	Some Cohomology	219
3.13	The Theorem of Weyl	222
3.14	The Levi Decomposition	224
3.15	The Analytic Group of a Lie Algebra	228
3.16	Reductive Lie Algebras	230
3.17	The Theorem of Ado	233
3.18	Some Global Results	238
	Exercises	247

Chapter 4 Complex Semisimple Lie Algebras And Lie Groups: Structure and Representation

260

4.1	Cartan Subalgebras	260
4.2	The Representations of $\mathfrak{sl}(2, \mathbb{C})$.	267
4.3	Structure Theory	273
4.4	The Classical Lie Algebras	293
4.5	Determination of the Simple Lie Algebras over \mathbb{C}	305
4.6	Representations with a Highest Weight	313
4.7	Representations of Semisimple Lie Algebras	324
4.8	Construction of a Semisimple Lie Algebra from its Cartan Matrix	329

4.9	The Algebra of Invariant Polynomials on a Semisimple Lie Algebra	333	
4.10	Infinitesimal Characters	337	
4.11	Compact and Complex Semisimple Lie Groups		342
4.12	Maximal Tori of Compact Semisimple Groups		351
4.13	An Integral Formula	356	
4.14	The Character Formula of H. Weyl	364	
4.15	Appendix. Finite Reflection Groups	369	
	Exercises	387	

Bibliography	417
---------------------	------------

Index	421
--------------	------------