

Contents

Preface	ix
Glossary of Mathematical Notation	xi
PART I FOUNDATIONS OF MECHANICS	
<i>This part is an extension of joint work with G. Caratú</i>	
Chapter 1 Introduction to Part I.	3
Chapter 2 A Digression on Manifolds and Diffeomorphisms	7
Chapter 3 Construction of Q : From Observables to the Configuration Space Manifold.	14
3.1 Construction of the configuration space Q	15
3.2 Topological and differential nature of Q	17
3.3 Invariants of a system under Q transformations	20
Chapter 4 Time and Transformations on Time.	24
Chapter 5 A Digression on Calculus on Manifolds, Bundles, Vector Fields and Forms.	28
5.1 Bundles.	28
5.2 Vector fields	34
5.3 Forms	39
5.4 The Lie derivative	43
5.5 Vector fields as generators of transformations	47
Chapter 6 From Trajectories to the Vector Field	49
Chapter 7 Lifting to a Carrier Space: Canonical Lifting.	64
Chapter 8 A Digression on Submanifolds and Smooth Maps.	78

8.1	Submanifolds from imbeddings	79
8.2	Submanifolds from submersions	83
Chapter 9	Transformations on TQ	85
Chapter 10	Integrating the Dynamics on TQ . Hamiltonian and Lagrangian Formalisms	90
10.1	Integrating the dynamics	90
10.2	Simple reduction of Hamiltonian dynamics.	97
10.3	Existence of symplectic forms on TQ	104
10.4	The Lagrangian formalism	107
10.5	Other classes of dynamical systems	109
Appendix.	112
Chapter 11	From the Tangent Bundle to the Cotangent Bundle.	115
11.1	The fiber derivative	115
11.2	Lifting vector fields from Q to T^*Q	120
Chapter 12	The Canonical Hamiltonian Formalism on T^*Q	122
12.1	Dynamics on T^*Q	122
12.2	Examples	124
12.3	Functions in involution.	127
Chapter 13	Equivalent Lagrangians and Hamiltonians	128
13.1	Ambiguity on TQ	128
13.2	Ambiguity on T^*Q	133
Chapter 14	Other Carrier Spaces: Action-Angle Variables and the Hamilton–Jacobi Method.	137
14.1	Action-angle variables	138
14.2	The Hamilton–Jacobi method.	144
Chapter 15	The Noether Theorem	151
15.1	The Hamiltonian formalism	151
15.2	The Lagrangian formalism	153
15.3	On the converse of the Noether theorem	161
Appendix to Part I:	Time-dependent Hamilton dynamics.	166
PART II REDUCTION, ACTIONS OF ALGEBRAS AND GROUPS		
Chapter 16	Introduction to Part II	173
SECTION A REDUCTION		
Chapter 17	Linear Dynamical Systems: A Prelude to Reduction.	177

17.1	Reduction by a constant of the motion (Hamiltonian system)	177
17.2	Simpler reduction of the same system.	181
17.3	Reduction of linear systems	182
Chapter 18	A Digression on Foliations and Distributions	186
18.1	Foliations	186
18.2	Distributions	194
18.3	Foliations and additional structure	199
Chapter 19	Reduction of Dynamical Systems through Regular Foliations	203
19.1	Projectability	203
19.2	Reduction through foliation	209
19.3	Remarks on integrating the dynamics on M	212
Chapter 20	Foliation of Symplectic Manifolds and Reduction of Hamiltonian Systems	219
20.1	Distributions on symplectic manifolds, function groups	219
20.2	Foliations generated by submersions	221
20.3	Reduction of Hamiltonian systems.	227
SECTION B ALGEBRA AND GROUP ACTIONS		
Chapter 21	A Digression on Lie Algebras and their Actions on Manifolds	235
21.1	Introduction: distributions and Lie algebras	235
21.2	Lie algebras.	238
21.3	Actions of Lie algebras on manifolds	241
21.4	The connection between Lie algebra actions and dynamics	249
Chapter 22	Actions of Lie Algebras on Symplectic Manifolds.	252
22.1	Symplectic, Hamiltonian, and other actions.	253
22.2	Introduction to the momentum map	258
22.3	The momentum map: details and examples.	262
22.4	The existence of Hamiltonian actions.	268
22.5	Symplectic forms on \mathfrak{G}^* and Poisson brackets	272
22.6	A detailed example: the Kepler problem.	273
Chapter 23	A Digression on Lie Groups and their Actions on Manifolds	277
23.1	Lie groups	278
23.2	Foliations and subgroups of $GL(n, \mathbb{R})$.	282
23.3	Lie groups and Lie algebras	285
23.4	Actions of Lie groups on manifolds	287
23.5	Relation between the actions of Lie groups and the actions of Lie algebras	291

23.6	Examples of Lie group actions	294
Chapter 24	Actions of Lie Groups on Symplectic Manifolds	304
24.1	Lie group actions which preserve differential forms	304
24.2	Reduction with symplectic actions: the momentum map	309
24.3	Reduction on T^*Q . Dynamics	312
Chapter 25	Parallelizable Manifolds. Dynamics on Lie Groups	317
25.1	Calculus on parallelizable manifolds	317
25.2	Explicit construction of parallelizations	319
25.3	Formalisms of dynamics of TG and T^*G	323
25.4	The rigid rotator	325
Chapter 26	Examples and Applications	330
26.1	The harmonic oscillator in n degrees of freedom	330
26.2	The nonrelativistic spinning particle	334
26.3	The electron–monopole system	336
26.4	Relativistic interacting particles	339
26.5	Action-angle variables and Hamilton–Jacobi: a second look with Lagrangian foliations	344
26.6	Uniqueness of the Lagrangian	349
26.7	Dynamical systems with constraints	352
Conclusion	355
References	358
Further reading	362
Index	371

