

Contents

Chapter 1 Countably Multinormed Spaces, Countable Union Spaces, and Their Duals

1.1. Introduction	1
1.2. Notations and Terminology	1
1.3. Linear Spaces	3
1.4. Sequential-Convergence Spaces	5
1.5. Seminorms and Multinorms	7
1.6. Multinormed Spaces	8
1.7. Countable-Union Spaces	14
1.8. Duals of Countably Multinormed Spaces	17
1.9. Duals of Countable-Union Spaces	24
1.10. Operators and Adjoint Operators	25

Chapter 2 Distributions and Generalized Functions

2.1. Introduction	32
2.2. The Spaces $\mathcal{D}_K(I)$, $\mathcal{D}(I)$, and Their Duals; Distributions.	32
2.3. The Space $\mathcal{E}(I)$ and Its Dual; Distributions of Compact Support	36
2.4. Generalized Functions.	38
2.5. Linear Partial Differential Operators Acting on Generalized Functions.	41
2.6. Generalized Functions That Depend Upon a Parameter and Parametric Differentiation	44
2.7. Generalized Functions That Are Concentrated Compact Sets	45

Chapter 3 The Two-Sided Laplace Transformation

3.1. Introduction	47
3.2. The Testing-Function Spaces $\mathcal{L}_{a,b}$ and $\mathcal{L}(w, z)$ and Their Duals	48

3.3.	The Two-Sided Laplace Transformation	55
3.4.	Operation-Transform Formulas	61
3.5.	Inversion and Uniqueness	64
3.6.	Characterization of Laplace Transforms and an Operational Calculus	70
3.7.	Convolution	73
3.8.	The Laplace Transformation of Convolution	78
3.9.	The Cauchy Problem for the Wave Equation in One-Dimensional Space	80
3.10.	The Right-Sided Laplace Transformation	84
3.11.	The n -Dimensional Laplace Transformation	90
3.12.	The Inhomogeneous Wave Equation in One-Dimensional Space	100

Chapter 4 The Mellin Transformation

4.1.	Introduction	102
4.2.	The Testing-Function Spaces $\mathcal{M}_{a,b}$ and $\mathcal{M}(w, z)$ and Their Duals	103
4.3.	The Mellin Transformation	106
4.4.	Operation-Transform Formulas	110
4.5.	An Operational Calculus for Euler Differential Equations	113
4.6.	Mellin-Type Convolution	116
4.7.	Dirichlet's Problem for a Wedge with a Generalized- Function Boundary Condition	121

Chapter 5 The Hankel Transformation

5.1.	Introduction	127
5.2.	The Testing-Function Space \mathcal{H}_μ and Its Dual	129
5.3.	Some Operations of \mathcal{H}_μ and \mathcal{H}'_μ	134
5.4.	The Conventional Hankel Transformation on \mathcal{H}_μ	139
5.5.	The Generalized Hankel Transformation	141
5.6.	The Hankel Transformation on $\mathcal{E}'(I)$	145
5.7.	An Operational Calculus	150
5.8.	A Dirichlet Problem in Cylindrical Coordinates	154
5.9.	A Cauchy Problem for Cylindrical Waves	157
5.10.	Hankel Transforms of Arbitrary Order	163
5.11.	Hankel Transforms of Certain Generalized Functions of Arbitrary Growth	168

Chapter 6 The K Transformation

6.1.	Introduction	170
6.2.	Some Classical Results	172
6.3.	The Testing-Function Space $\mathcal{K}_{\mu,a}$ and Its Dual	176

6.4.	The K Transformation	181
6.5.	The Analyticity of a K Transform	184
6.6.	Inversion	186
6.7.	Characterization of K Transforms	195
6.8.	An Operational Calculus	198
6.9.	Applications to Certain Time-Varying Electrical Networks	200

Chapter 7 The Weierstrass Transformation

7.1.	Introduction	205
7.2.	The Testing-Function Spaces $\mathcal{W}_{a,b}$ and $\mathcal{W}(w, z)$ and Their Duals	206
7.3.	The Weierstrass Transformation	210
7.4.	Another Inversion Formula	215
7.5.	The Cauchy Problem for the Heat Equation for One-Dimensional Flow	223

Chapter 8 The Convolution Transformation

8.1.	Introduction	229
8.2.	Convolution Kernels	230
8.3.	The Convolution Transformation	234
8.4.	Inversion	236
8.5.	The One-Sided Laplace Transformation	241
8.6.	The Stieltjes Transformation	244

Chapter 9 Transformations Arising from Orthonormal Series Expansions

9.1.	Introduction	247
9.2.	The Space $L_2(I)$	248
9.3.	The Testing-Function Space \mathcal{A}	252
9.4.	The Generalized-Function Space \mathcal{A}'	257
9.5.	Orthonormal Series Expansions and Generalized Integral Transformations	259
9.6.	Characterizations of the Generalized Functions in \mathcal{A}' and Their Transforms	260
9.7.	An Operational Calculus for \mathfrak{N}	264
9.8.	Particular Cases	265
9.9.	Application of the Finite Fourier Transformation: A Dirichlet Problem for a Semi-Infinite Channel	273
9.10.	Applications of the Laguerre and Jacobi Transfor- mations: The Time-Domain Synthesis of Signals	275
9.11.	Application of the Legendre Transformation: A Dirichlet Problem for the Interior of the Unit Sphere	278

9.12. Application of the Finite Hankel Transformation of the First Form: A Dirichlet Problem for a Semi-Infinite Cylinder	281
9.13. Application of the Finite Hankel Transformation of the Second Form: Heat Flow in an Infinite Cylinder with a Radiation Condition	284
Bibliography	287
Index of Symbols	295
Index	297

