



# Contents

Foreword . . . . .	vii
Introduction . . . . .	1
Reader's Guide . . . . .	7
Historical Notes . . . . .	9

## Part 1. Hysteresis Operators

Chapter I. Genesis of Hysteresis . . . . .	12
1. What is Hysteresis? . . . . .	12
2. Continuous Hysteresis . . . . .	14
3. Catastrophic Hysteresis . . . . .	17
4. Mean Field Theory of Ferromagnetism . . . . .	20
5. Landau's Theory of Phase Transitions . . . . .	23
6. Comments . . . . .	29
Chapter II. Rheological and Circuital Models . . . . .	32
1. Rheological Models . . . . .	32
2. Elementary Rheological Models . . . . .	35
3. Models of Damage . . . . .	42
4. Circuital Models . . . . .	49
5. Comments . . . . .	58
Chapter III. Plays, Stops and Prandtl-Ishlinskii Models . . . . .	59
1. Hysteresis Operators . . . . .	59
2. Plays . . . . .	63
3. Stops . . . . .	75
4. Prandtl-Ishlinskii Models . . . . .	83
5. Damage Operator . . . . .	88
6. Memory Sequences . . . . .	90
7. Comments . . . . .	94
Chapter IV. The Preisach Model . . . . .	97
1. The Preisach Operator . . . . .	97
2. Geometry of the Preisach Plane . . . . .	103

3. Continuity Properties of the Preisach Operator . . . . .	112
4. Inverse Preisach Operator . . . . .	119
5. Vectorial Preisach Models . . . . .	122
6. Hysteresis and Laziness . . . . .	126
7. Comments . . . . .	128
<b>Chapter V. The Duhem Model . . . . .</b>	<b>130</b>
1. Standard Duhem Model . . . . .	131
2. Generalized Duhem Model . . . . .	141
3. Vectorial Duhem Model . . . . .	145
4. Comparison of Hysteresis Models . . . . .	147
5. Comments . . . . .	149
<b>Chapter VI. Discontinuous Hysteresis . . . . .</b>	<b>151</b>
1. Completed Delayed Relay Operator . . . . .	151
2. Weak Formulation of Delayed Relay Operators . . . . .	161
3. Double Inclusions and Hysteresis . . . . .	168
4. Differential Plays and Differential Stops . . . . .	173
5. Discontinuous Hysteresis with Vector Input . . . . .	176
6. Comments . . . . .	184

## **Part 2. Partial Differential Equations with Hysteresis**

<b>Chapter VII. P.D.E. Models of Elasto-Plasticity . . . . .</b>	<b>186</b>
1. Prandtl-Ishlinskii Operator of Stop-Type . . . . .	186
2. Prandtl-Ishlinskii Operator of Play-Type . . . . .	197
3. A Problem in Elasto-Visco-Plasticity . . . . .	204
4. Comments . . . . .	208
<b>Chapter VIII. Hysteresis and Semigroups . . . . .</b>	<b>211</b>
1. Some Equations with Hysteresis . . . . .	212
2. Accretive Operators Related to Hysteresis . . . . .	225
3. Space-Structured Systems . . . . .	233
4. Another Class of Accretive Operators . . . . .	241
5. Order and Approximation Properties . . . . .	247
6. Application to Hysteresis Models . . . . .	250
7. Comments . . . . .	255
<b>Chapter IX. Quasilinear P.D.E.s with Memory . . . . .</b>	<b>257</b>
1. Quasilinear Parabolic Equations with Memory . . . . .	258
2. Quasilinear Parabolic Equations with Hysteresis . . . . .	272
3. Other Quasilinear Equations . . . . .	280
4. Asymptotic Results . . . . .	287
5. Krejčí's Estimate . . . . .	290
6. Comments . . . . .	292

<b>Chapter X. Semilinear P.D.E.s with Memory . . . . .</b>	<b>295</b>
1. Semilinear Parabolic Equations with Memory . . . . .	296
2. Other Semilinear Equations . . . . .	305
3. Minimum and Maximum Solutions . . . . .	312
4. First Order Hyperbolic Equations with Memory . . . . .	316
5. Travelling Waves . . . . .	320
6. Comments . . . . .	324
<b>Chapter XI. P.D.E.s with Discontinuous Hysteresis . . . . .</b>	<b>325</b>
1. Genesis of O.D.E.s with Hysteresis . . . . .	326
2. Genesis of P.D.E.s with Hysteresis . . . . .	330
3. Quasilinear Parabolic P.D.E.s with Discontinuous Hysteresis . . . . .	334
4. Another Estimation Procedure . . . . .	339
5. Semilinear Parabolic P.D.E.s with Discontinuous Hysteresis . . . . .	343
6. Quasilinear Hyperbolic Equations with Hysteresis . . . . .	346
7. Homogenization of the Preisach Model . . . . .	353
8. A Biological Problem with Hysteresis . . . . .	356
9. Summary of Results for P.D.E.s with Memory . . . . .	357
10. Comments . . . . .	359
 <b>Appendix</b>	
<b>Chapter XII. Some Tools . . . . .</b>	<b>362</b>
1. Some Function Spaces . . . . .	362
2. Spaces of Functions with Values in Banach Spaces . . . . .	363
3. Some Spaces of Operators . . . . .	369
4. Nonlinear Semigroups of Contractions . . . . .	370
5. Elements of Convex Analysis . . . . .	374
6. Order Preserving Mappings . . . . .	378
7. Strict Convergence in $BV(0,T)$ . . . . .	381
8. Comments . . . . .	388
Conclusion . . . . .	389
<b>Bibliography . . . . .</b>	<b>391</b>