Table of Contents

AUTHOR'S PREFACE v

TRANSLATOR'S PREFACE vi

TERMINOLOGY vii

INTRODUCTION 1

I: ANALYSIS IN LINEAR SPACES 8
1. Various forms of continuity for operators 8
2. The Stieltjes integral and curvilinear integrals 24
3. The differential and derivative of an operator 35
4. On the continuity and compactness of the derivative 43

II: POTENTIAL OPERATORS 54
5. The gradient of a functional, potential operators, and conditions for the potentialness of a differentiable operator 54
6. Conditions for the potentialness of non-differentiable operators 60
7. On the compactness and strong continuity of potential operators 64

III: OPERATOR EQUATIONS AND THE EXTREME POINTS OF FUNCTIONALS 73
8. On the weak semi-continuity of functionals 73
9. The extreme points of a functional 77
10. On the solvability of operator equations 80

IV: CONDITIONALLY EXTREME AND CONDITIONALLY CRITICAL POINTS OF A FUNCTIONAL 89
11. Second order surfaces in Hilbert space 89

12. Conditionally extreme and conditionally critical points of a functional 96
13. On conditionally extreme points and critical points 100
14. Conditionally critical points of even functionals 111

V: PROPER FUNCTIONS AND BRANCH POINTS OF NONLINEAR OPERATORS 121
15. Proper functions of a product of positive and potential operators 121
16. Proper functions of the product of quasi-definite and potential operators 130
17. Branch points of nonlinear operators 138

VI: OPERATORS AND FUNCTIONALS OF SPECIAL TYPE 147
18. A type of S-property for functions 147
19. On the continuity of the Nemytsky operator 154
20. On the differentiability of the Nemytsky and Hammerstein operators 163
21. The potentials of the Hammerstein and Liapunov-Lichtenstein operators 173
22. On certain properties of integral quadratic forms in the spaces $L^q (q \leq 2)$ 184
23. The functional $f(\xi)$ 192

VII: NONLINEAR INTEGRAL EQUATIONS 204
24. The existence and uniqueness of solutions 204
25. Proper functions of nonlinear integral operators 222
26. Proper numbers and branch points of nonlinear integral operators 241

VIII: NEWTON'S METHOD 258
27. Equations of the form $P(x) = \theta$ 258
28. Corollaries of the theorem on the convergence of Newton's method 271
29. The application of Newton's method to concrete functional equations 279

SUPPLEMENT 299

BIBLIOGRAPHY 311

INDEX 321