

Contents

Preface	xii
Introduction	xiv

Chapter 1

Systems of implicit functions and the classical branching theory

§1. The implicit function problem	1
1.1. Classical implicit function theorems (1). 1.2. General analysis of the implicit function problem (2). 1.3. The analytic case (7).	
§2. The one-dimensional branching case and Newton's diagram	9
2.1. Newton's diagram (10). 2.2. Properties of the solutions (12). 2.3. Examples (16). 2.4. Investigation of the branching equations. The case of simple roots of the defining equation (18). 2.5. The case of multiple roots of the defining equation (22). 2.6. Real solutions (24). 2.7. Special configurations of Newton's diagram (26).	

Chapter 2

Investigation of the branching equation in the many-dimensional case

§3. Transformation of the branching equation	34
3.1. Reduction to regular form (34). 3.2. Reduction to normal form (37).	
§4. Topics in divisibility theory	41
4.1. The ring of power series (41). 4.2. Common divisors and an analog of the Euclidean algorithm (44). 4.3. A primitive GCD (48). 4.4. Application to distinguished polynomials (52).	

§5.	The two-dimensional branching case	52
	5.1. General investigation of the problem (53). 5.2. Branching indices in the two-dimensional case (56). 5.3. Special cases (59).	
§6.	The many-dimensional branching case	70
	6.1. The Kronecker elimination method (70). 6.2. Small solutions of the branching equation and the method of elimination (74). 6.3. The quasi-regular branching case (76). 6.4. The degenerate branching case (78). 6.5. Isolated zero solution (79).	

Chapter 3

The branching equation for non-linear integral and integro-differential equations

§7.	The Lyapunov–Schmidt integral equations	80
	7.1. Integro-power series of one functional argument (80). 7.2. Integro-power series of many functional arguments (82). 7.3. Integro-power series of integro-power series (83). 7.4. Contraction property of the Lyapunov–Schmidt operator (84). 7.5. The simple equation (87). 7.6. The Lichtenstein successive approximations (89). 7.7. Application of the method of majorants (91). 7.8. The simple equation in the case of uniform convergence (96).	
§8.	The general Lyapunov–Schmidt integral equation	99
	8.1. The regular case (100). 8.2. The regular case with many arguments (102). 8.3. Schmidt’s lemma (103). 8.4. The one-dimensional branching case (104). 8.5. The many-dimensional branching case (108). 8.6. Possible generalizations (111).	
§9.	Lyapunov–Schmidt systems of equations and certain integro-differential equations	112
	9.1. Lyapunov–Schmidt systems of two equations (113). 9.2. The regular case (115). 9.3. The branching case (118). 9.4. Some non-linear integro-differential equations of first order (123). 9.5. Other types of integro-differential equations (126). 9.6. A different method for constructing the branching equation (130).	

Chapter 4

General integral equation and the coefficients of the branching equation

§10. The general integral equation	134
10.1. Formulation of the problem and preliminary remarks (134). 10.2. The regular case (136). 10.3. The one-dimensional branching case (138). 10.4. The many-dimensional branching case (144). 10.5. A special case (152). 10.6. The Hammerstein equation (159).	
§11. The coefficients of the branching equations	167
11.1. The coefficients of the one-dimensional branching equation for the general non-linear integral equation (168). 11.2. The coefficients of the one-dimensional branching equation for the non-linear Hammerstein integral equation (178). 11.3. The coefficients of the two-dimensional branching equation for the general non-linear integral equation (180). 11.4. The coefficients of the two-dimensional branching equation for special cases (187). 11.5. Some properties of the coefficients of the branching equation (198).	

Chapter 5

Characterization and construction of solutions of non-linear equations

§12. Characterization of solutions of non-linear integral equations	201
12.1. Preliminary remarks (201). 12.2. Characterization of solutions for the one-dimensional branching case (205). 12.3. Bifurcation points in the one-dimensional branching case (209). 12.4. Characterization of solutions in the two-dimensional branching case (212). 12.5. Characterization of solutions in the many-dimensional branching case (214). 12.6. Branching of isolated solutions (214). 12.7. Bifurcation points in the many-dimensional branching case (220).	
§13. Construction of solutions to non-linear integral equations	222
13.1. Methods for constructing solutions (222). 13.2. The one-dimensional branching case (224). 13.3. The two-dimensional branching case (228). 13.4. The Nekrasov equation (233). 13.5. The branching equation for Nekrasov's equation (235).	

§14. Singular solutions to non-linear integral equations	242
14.1. Formulation of the problem (242). 14.2. Reduction to the problem of small solutions (242). 14.3. Singular solutions in the regular case (243). 14.4. Investigation of the auxiliary equation (245). 14.5. Branching of the solutions of the fundamental equation (247). 14.6. Singular solutions in Lebesgue spaces (249).	

Chapter 6

Branching of periodic solutions of differential equations

§15. Periodic solutions of non-autonomous systems	251
15.1. Formulation of the problem (251). 15.2. The Poincaré method (252). 15.3. Periodicity conditions and the branching equation (253). 15.4. Characterization of solutions in the regular case (254). 15.5. Characterization of solutions in the one-dimensional branching case (256). 15.6. Characterization of solutions in the many-dimensional branching case (257).	
§16. Periodic solutions of quasi-linear systems	258
16.1. Formulation of the problem (258). 16.2. The periodicity condition (260). 16.3. Derivation of the branching equation (261). 16.4. Characterization of solutions and additional remarks (262).	
§17. Periodic solutions of autonomous systems	263
17.1. The Poincaré problem for autonomous systems (263). 17.2. The branching equation (264). 17.3. The main results (266). 17.4. The method of undetermined coefficients (267). 17.5. Autonomous systems with one degree of freedom (269).	
§18. Examples	272
18.1. Non-autonomous systems with one degree of freedom (272). 18.2. Autonomous systems with one degree of freedom (282).	
§19. Additional problems involving periodic solutions	283
19.1. Singular periodic solutions of non-autonomous systems (283). 19.2. Branching of periodic solutions in Banach spaces (285).	
§20. On the stability of periodic solutions dependent on a small parameter	290
20.1. Preliminary information from the Lyapunov stability theory (290). 20.2. Stability of the solutions of the Poincaré problem (292).	

Non-linear equations in Banach spaces

§21. Some topics in the theory of linear operators in Banach spaces	306
21.1. Fredholm operators (306). 21.2. Special decompositions of spaces into direct sums of subspaces (307). 21.3. Restriction of an operator and the generalized Schmidt lemma (309). 21.4. The relation to adjoint operators (311). 21.5. Unbounded Fredholm operators (313).	
§22. Power operators, Taylor series, the implicit operator theorems	314
22.1. Power operators (314). 22.2. Power series (316). 22.3. Implicit operator theorems (319).	
§23. The Lyapunov–Schmidt branching equation	323
23.1. Formulation of the problem (323). 23.2. Derivation of the branching equation by restriction of the operator (325). 23.3. Derivation of the branching equation with the aid of Schmidt's lemma (327). 23.4. A fundamental theorem on the branching equation (329). 23.5. The branching equation in the analytic case (329). 23.6. The branching equation for unbounded operators (330).	
§24. Investigation of the one-dimensional branching case	331
24.1. Calculation of the leading coefficients (331). 24.2. The degenerate case (334). 24.3. The quasi-regular case (Problem A) (335). 24.4. Problem B (the non-degenerate case) (228). 24.5. The real case (339). 24.6. The branching of solutions of equations with sufficiently smooth operators (340). 24.7. The case of a functional parameter. Series in powers of homogeneous operators (341). 24.8. The case of two numerical parameters (345).	
§25. The many-dimensional branching case	347
25.1. Transformation to an equivalent system (347). 25.2. The coefficients of the two-dimensional branching equation (349). 25.3. The two-dimensional branching case (350). 25.4. The general case (351). 25.5. The branching of isolated solutions (351). 25.6. Bifurcation points (352).	

Chapter 8

Branching of the solutions of non-linear equations in the singular case

- §26. Noether operators 354
26.1. Noether operators (354). 26.2. Decomposition of spaces into a direct sum of subspaces. Restriction of an operator (355). 26.3. Atkinson's theorem. The relation to adjoint operators (356).
- §27. Theorems on the branching of solutions 360
27.1. Formulation of the problem (360). 27.2. The case $n > 0$, $m = 0$ (360). 27.3. The case $n = 0$, $m > 0$ (362). 27.4. The fundamental case (362). 27.5. Branching of solutions of an equation with an unbounded operator (363).
- §28. Branching of solutions of non-linear singular integral equations 364
28.1. Linear singular integral operators with a Cauchy kernel in Hölder spaces (364). 28.2. Non-linear singular integral equations with a Cauchy kernel in Hölder spaces (366). 28.3. The analytic case (368). 28.4. Non-linear singular integral equations with a Hilbert kernel in Lebesgue spaces (370).
- §29. Branching of solutions of boundary-value problems for non-linear elliptic equations 375
29.1. Boundary-value problems for second-order elliptic equations in Hölder spaces (375). 29.2. Boundary-value problems in the plane for k -th order elliptic systems in a Hölder space (380). 29.3. Boundary-value problems for elliptic equations in the space of summable functions (382).

Chapter 9

Selected problems in perturbation theory

- §30. Jordan chains and systems of Fredholm operators 387
30.1. A -Jordan chains for $n = 1$ (387). 30.2. A -Jordan chains and sets where $n > 1$ (389). 30.3. Conditions for the completeness of an A -Jordan set (391). 30.4. Example (395).
- §31. Perturbation of a linear equation by a small linear term 396
31.1. The case $n = 1$ (396). 31.2. The case $n > 1$ (399). 31.3. The method of undetermined coefficients (400).

§32. Branching of eigenvalues and eigenelements of Fredholm operators	404
32.1. Derivation of the branching equation (404). 32.2. The branching equation in the analytical case (406). 32.3. The degenerate and the non-degenerate case (407). 32.4. The one-dimensional case (409). 32.5. The method of undetermined coefficients (413). 32.6. The many-dimensional case (418).	

§33. Singular solutions of non-linear equations	424
33.1. Statement of the problem and basic concepts (424). 33.2. Perturbation of a linear equation by a small non-linear term (426). 33.3. Generalized Jordan chain (428). 33.4. Characteristics of certain polynomials (430). 33.5. The fundamental case of the perturbation problem (433). 33.6. Second-order perturbations (441).	

Chapter 10

Applied problems

§34. Small bending deformation of a straight rod under constant load	450
§35. The theory of small deflections in elastic plates	454
35.1. Formulation of the problem and general remarks (454). 35.2. A circular plate (457). 35.3. Investigation of the branching equation (461). 35.4. A circular plate with zero external load (464).	

§36. Oscillations of a satellite in the plane of an elliptic orbit	465
--	-----

§37. Standing waves	469
37.1. Formulation of the problem and derivation of the fundamental integral equations (469). 37.2. Reduction to a system (473). 37.3. The regular case (474). 37.4. The branching case (475).	

Bibliography	478
------------------------	-----

Subject index	486
-------------------------	-----