

Contents

I. Volterra Equations	1
1·1. A Mechanical Problem Leading to an Integral Equation	1
1·2. Integral Equations and Algebraic Systems of Linear Equations	3
1·3. Volterra Equations	5
1·4. L_2 -Kernels and Functions	8
1·5. Solution of Volterra Integral Equations of the Second Kind	10
1·6. Volterra Equations of the First Kind	15
1·7. An Example	17
1·8. Volterra Integral Equations and Linear Differential Equations	18
1·9. Equations of the Faltung Type (Closed Cycle Type) . .	22
1·10. Transverse Oscillations of a Bar	26
1·11. Application to the Bessel Functions	32
1·12. Some Generalizations of the Theory of Volterra Equations	38
1·13. Non-Linear Volterra Equations	42
II. Fredholm Equations	49
2·1. Solution by the Method of Successive Approximations: Neumann's Series	49
2·2. An Example	53
2·3. Fredholm's Equations with Pincherle-Goursat Kernels	55
2·4. The Fredholm Theorem for General Kernels	64
2·5. The Formulae of Fredholm	66
2·6. Numerical Solution of Integral Equations	75
2·7. The Fredholm Solution of the Dirichlet Problem. . .	76
III. Symmetric Kernels and Orthogonal Systems of Functions	81
3·1. Introductory Remarks and a Process of Orthogonali- zation	81
3·2. Approximation and Convergence in the Mean	83
3·3. The Riesz-Fischer Theorem	88
3·4. Completeness and Closure	90
3·5. Completeness of the Trigonometric System and of the Polynomials	95
3·6. Approximation of a General L_2 -Kernel by Means of PG-Kernels	98
3·7. Enskog's Method	100
3·8. The Spectrum of a Symmetric Kernel	102

3·9. The Bilinear Formula	106
3·10. The Hilbert-Schmidt Theorem and Its Applications	110
3·11. Extremal Properties and Bounds for Eigenvalues	118
3·12. Positive Kernels—Mercer's Theorem	124
3·13. Connection with the Theory of Linear Differential Equations	127
3·14. Critical Velocities of a Rotating Shaft and Transverse Oscillations of a Beam	136
3·15. Symmetric Fredholm Equations of the First Kind	143
3·16. Reduction of a Fredholm Equation to a Similar One with a Symmetric Kernel	145
3·17. Some Generalizations	150
3·18. Vibrations of a Membrane	154
IV. Some Types of Singular or Non-Linear Integral Equations	161
4·1. Orientation and Examples	161
4·2. Equations with Cauchy's Principal Value of an Integral and Hilbert's Transformation	166
4·3. The Finite Hilbert Transformation and the Airfoil Equation	173
4·4. Singular Equations of the Carleman Type	185
4·5. General Remarks About Non-Linear Integral Equations	197
4·6. Non-Linear Equations of the Hammerstein Type	202
4·7. Forced Oscillations of Finite Amplitude	213
Appendix I. Algebraic Systems of Linear Equations	219
Appendix II. Hadamard's Theorem	223
Exercises	227
References	231
Index	235

