CONTENTS

Preface v

CHAPTER 1. ORDINARY DIFFERENTIAL EQUATIONS IN MORE THAN TWO VARIABLES 1

1. Surfaces and Curves in Three Dimensions 1
2. Simultaneous Differential Equations of the First Order and the First Degree in Three Variables 7
3. Methods of Solution of $dx/P = dy/Q = dz/R$ 10
4. Orthogonal Trajectories of a System of Curves on a Surface . . . 15
5. Pfaffian Differential Forms and Equations 18
6. Solution of Pfaffian Differential Equations in Three Variables . . . 26
7. Carathéodory's Theorem 33
8. Application to Thermodynamics 39

Miscellaneous Problems 42

CHAPTER 2. PARTIAL DIFFERENTIAL EQUATIONS OF THE FIRST ORDER . 44

1. Partial Differential Equations 44
2. Origins of First-order Partial Differential Equations 45
3. Cauchy's Problem for First-order Equations 47
4. Linear Equations of the First Order 49
5. Integral Surfaces Passing through a Given Curve 56
6. Surfaces Orthogonal to a Given System of Surfaces 57
7. Nonlinear Partial Differential Equations of the First Order 59
8. Cauchy's Method of Characteristics 61
9. Compatible Systems of First-order Equations 67
10. Charpit's Method 69
11. Special Types of First-order Equations 70
12. Solutions Satisfying Given Conditions 73
13. Jacobi's Method 78
14. Applications of First-order Equations 81

Miscellaneous Problems 85

CHAPTER 3. PARTIAL DIFFERENTIAL EQUATIONS OF THE SECOND ORDER 88

1. The Origin of Second-order Equations 88
2. Second-order Equations in Physics 90
3. Higher-order Equations in Physics 94

4. Linear Partial Differential Equations with Constant Coefficients . 96
5. Equations with Variable Coefficients 105
6. Characteristic Curves of Second-order Equations 110
7. Characteristics of Equations in Three Variables 115
8. The Solution of Linear Hyperbolic Equations 119
9. Separation of Variables 123
10. The Method of Integral Transforms 126
11. Nonlinear Equations of the Second Order 131
Miscellaneous Problems 136

CHAPTER 4. LAPLACE'S EQUATION 141

1. The Occurrence of Laplace's Equation in Physics 141
2. Elementary Solutions of Laplace's Equation 145
3. Families of Equipotential Surfaces 148
4. Boundary Value Problems 151
5. Separation of Variables 156
6. Problems with Axial Symmetry 161
7. Kelvin's Inversion Theorem 164
8. The Theory of Green's Function for Laplace's Equation 167
9. The Relation of Dirichlet's Problem to the Calculus of Variations . 174
10. "Mixed" Boundary Value Problems 175
11. The Two-dimensional Laplace Equation 180
12. Relation of the Logarithmic Potential to the Theory of Functions . 184
13. Green's Function for the Two-dimensional Equation 193
Miscellaneous Problems 197

CHAPTER 5. THE WAVE EQUATION 209

1. The Occurrence of the Wave Equation in Physics 209
2. Elementary Solutions of the One-dimensional Wave Equation . . 215
3. The Riemann-Volterra Solution of the One-dimensional Wave Equation 221
4. Vibrating Membranes: Application of the Calculus of Variations . 226
5. Three-dimensional Problems 232
6. General Solutions of the Wave Equation 239
7. Green's Function for the Wave Equation 244
8. The Nonhomogeneous Wave Equation 249
9. Riesz's Integrals 254
10. The Propagation of Sound Waves of Finite Amplitude 257
Miscellaneous Problems 262

CHAPTER 6. THE DIFFUSION EQUATION 274

1. The Occurrence of the Diffusion Equation in Physics 274
2. The Resolution of Boundary Value Problems for the Diffusion Equation 278

3. Elementary Solutions of the Diffusion Equation 282
4. Separation of Variables 286
5. The Use of Integral Transforms 290
6. The Use of Green's Functions 294
7. The Diffusion Equation with Sources 299
Miscellaneous Problems 303

APPENDIX. SYSTEMS OF SURFACES 309

1. One-parameter Systems 309
2. Two-parameter Systems 311
3. The Edge of Regression 312
4. Ruled Surfaces 314

Solutions to the Odd-numbered Problems 315

Index 323