



# Contents

Preface

Introduction

Part one THE RESIDUE METHOD

Chapter 1 Dini's theorem generalised

1. Lagrange's formula and systems of integral equations 25
2. Generalised existence theorem 30
3. Existence theorems 33

Chapter 2 Asymptotic representations of solutions of linear differential equations with a complex parameter

1. Formal solutions of first-order systems 39
2. Asymptotic representations of solutions of a system of first-order equations 63
3. Asymptotic representations of solutions of a single equation of higher order 76

## Chapter 3 Expansion of vector-valued functions

1. Boundary-value problems for a system of first-order equations with piece-wise coefficients 84
2. Theorem on the expansion in series of residues of solutions of boundary-value problems with a parameter for systems of ordinary differential equations with discontinuous coefficients 130
3. Derivation of the solution of the spectral problem for a single equation of higher order with discontinuous coefficients 146

## Chapter 4 Solution of one-dimensional mixed problems for systems of equations with discontinuous coefficients

1. Mixed problems with boundary conditions containing time derivatives 152
2. Mixed problems with boundary conditions containing no time derivatives 178
3. The mixed problem with separate variables 198

## Chapter 5 Residue method for solving multi-dimensional mixed problems

1. Procedure for solving multi-dimensional mixed problems 207
2. Residue method of separating variables 211
3. Formula for expanding an arbitrary function in a series of residues of a solution of a certain class of multi-dimensional spectral problems 220
4. Problems in subterranean hydromechanics 225

## Part two THE CONTOUR-INTEGRAL METHOD

## Chapter 6 Contour-integral method of solving one-dimensional mixed problems for second-order equations with discontinuous coefficients

1. Equations containing only first-order time derivatives 237
2. Asymptotic representation of the solution of a spectral problem outside a  $\delta$ -neighbourhood of the spectrum 239

3. Solution of the mixed problem (6.1.1) - (6.1.3) with parabolicity in the sense of Petrovskiy	263
4. Expansion of an arbitrary function in a series of residues of the spectral problem: necessary and sufficient conditions for the correct formulation of problem (6.1.1) - (6.1.3)	293
5. Solution of mixed problems for equations containing first-order time derivatives: necessary and sufficient conditions	301
Chapter 7 Solution of one-dimensional mixed problems for linear differential equations with discontinuous coefficients and time-dependent boundary conditions	
1. Asymptotic representation of the solution of a spectral problem outside a $\delta$ -neighbourhood	320
2. Solution of mixed problems for equations containing only first-order time derivatives	345
Chapter 8 Solution of a multi-dimensional spectral problem for a single elliptic equation with a large complex parameter	
1. Fundamental solution and its bounds	357
2. Formulae for the saltus in the potentials of a single and a double layer	375
3. Solution of the spectral problem for a homogeneous equation and bounds for it	377
4. Bound for the regular part of the Green's function of the spectral problem	384
Chapter 9 Multi-dimensional mixed problem for a parabolic equation with time-dependent boundary conditions	
1. Mixed problem for a homogeneous equation with homogeneous initial condition	389
2. Mixed problem for a non-homogeneous equation with homogeneous boundary but non-homogeneous initial conditions	394
3. Actual solution of mixed problems	411

Chapter 10 Multi-dimensional mixed problem for parabolic equations with discontinuous coefficients and time-dependent boundary conditions	
1. A mixed problem and corresponding spectral problem	416
2. Fundamental solution of the spectral problem and a bound for its absolute value	418
3. Solution of the spectral problem and a bound for its absolute value	421
4. Solution of the mixed problem for equations with discontinuous coefficients	428
References	431
Index	438

