

Contents

PREFACE TO THE THIRD EDITION	xiii
From the Preface to the Second Edition	xiv
From the Preface to the First Edition	xiv
LIST OF NOTATIONS AND ABBREVIATIONS	xvii

1A. DIVIDED DIFFERENCES

Divided Differences for Distinct Arguments	1
Symmetry	2
Hermite's Integral Representation	3
Mean Value Formulas	5

1B. CONFLUENT CASE. INTERPOLATION

Confluent Divided Differences	8
Continuity of Confluent Divided Differences	9
Various Formulas for Divided Differences	10
Newton's Interpolation Formula	12
General Interpolation Problem	14
Polynomial Interpolation	15
The Remainder for a General Interpolating Function	15
Triangular Schemes for Computing Divided Differences	16

2. INVERSE INTERPOLATION. DERIVATIVES OF THE INVERSE FUNCTION. ONE INTERPOLATION POINT

The Concept of Inverse Interpolation	18
Darboux's Theorem on Values of $f'(x)$	19
Derivatives of the Inverse Function	20
One Interpolation Point	22
A Development of a Zero of $f(x)$	25

3. METHOD OF FALSE POSITION (REGULA FALSI)

Definition of the Regula Falsi	27
Use of Inverse Interpolation	28
Geometric Interpretation (Fourier's Conditions)	30
Iteration with Successive Adjacent Points	31
Horner Units and Efficiency Index	32
The Rounding-Off Rule	33
Locating the Zero with the Regula Falsi	34
Examples of Computation by the Regula Falsi	35

4. ITERATION

A Convergence Criterion for an Iteration	38
Points of Attraction and Repulsion	38
Improving the Convergence	40

5. FURTHER DISCUSSION OF ITERATIONS. MULTIPLE ZEROS

Iterations by Monotonic Iterating Functions	47
Multiple Zeros	48
Connection of the Regula Falsi with the Theory of Iteration.	51

6. THE NEWTON-RAPHSON METHOD

The Idea of the Newton-Raphson Method.	53
The Use of Inverse Interpolation	53
Comparison of Regula Falsi and Newton-Raphson Method.	55

7. FUNDAMENTAL EXISTENCE THEOREMS IN THE NEWTON-RAPHSON ITERATION

Error Estimates A Priori and A Posteriori	56
Fundamental Existence Theorems	56

8. AN ANALOG OF THE NEWTON-RAPHSON METHOD FOR MULTIPLE ROOTS

Convergence of Schröder's Iteration for Multiple Roots	61
Error Estimates A Priori	63
Recurrent Error Estimates	65
Evaluation of Exact Multiplicity	67

9.	FOURIER BOUNDS FOR THE NEWTON–RAPHSON ITERATION	69
10.	DANDELIN BOUNDS FOR THE NEWTON–RAPHSON ITERATION	73
11.	THREE INTERPOLATION POINTS	
	Interpolation by Linear Fractions	79
	Two Coincident Interpolation Points.	80
	Error Estimates	81
	Use in Iteration Procedure	82
12.	LINEAR DIFFERENCE EQUATIONS	
	Inhomogeneous and Homogeneous Difference Equations	84
	General Solution of the Homogeneous Equation.	85
	Lemma on Division of Power Series	86
	Asymptotic Behavior of Solutions of (12.1)	87
	Asymptotic Behavior of Errors in the Regula Falsi Iteration.	90
	A Theorem on Roots of Certain Equations	91
13.	n DISTINCT POINTS OF INTERPOLATION	
	Error Estimates	94
	Iteration with n Distinct Interpolation Points	95
	Discussion of the Roots of Some Special Equations	97
14.	$n+1$ COINCIDENT INTERPOLATION POINTS AND TAYLOR DEVELOPMENT OF THE ROOT	
	Statement of the Problem	103
	A Theorem on Inverse Functions and Conformal Mapping	103
	Theorem on the Error of the Taylor Approximation to the Root	106
	Discussion of the Conditions of the Theorem	107
15.	THE SQUARE ROOT ITERATION	
	Polynomials with Simple Real Zeros Only	110
	Modification for Multiple Zeros	113
	Differentiable Functions and Complex Zeros	115

16. FURTHER DISCUSSION OF SQUARE ROOT ITERATION	
Local Formulation of the Existence and Convergence Theorem	119
Extension to Entire Functions	124
17. A GENERAL THEOREM ON ZEROS OF INTERPOLATING POLYNOMIALS	127
18. APPROXIMATION OF EQUATIONS BY ALGEBRAIC EQUATIONS OF A GIVEN DEGREE. ASYMPTOTIC ERRORS FOR SIMPLE ROOTS	
Convergence of Zeros of Interpolating Polynomials	131
Asymptotic Errors for Simple Zeros	132
19. NORMS OF VECTORS AND MATRICES	
Vector Norms	135
Matrix Norms $ A _1$ and $ A _\infty$	137
Eigenvalues of A	140
20. TWO THEOREMS ON CONVERGENCE OF PRODUCTS OF MATRICES	143
21. A THEOREM ON DIVERGENCE OF PRODUCTS OF MATRICES	146
22. CHARACTERIZATION OF POINTS OF ATTRACTION AND REPULSION FOR ITERATIONS WITH SEVERAL VARIABLES	
Points of Attraction and Repulsion	150
An Example	153
23. EUCLIDEAN NORMS	
Euclidean Length and Frobenius Norm	155
Hermitian Matrices	156
Euclidean Norm of a Matrix	157

24. MINKOWSKI NORMS, $\Delta_p(A)$, $\Delta_{p,p'}(A)$

Minkowski Norms	159
$ A _p$ and $ A _{p,p'}$	159
$\Delta_{p,p'}(A)$ and $\Delta_p(A)$	161
Inequalities for $\Delta_{p,p'}(A)$	163
Variation of the Inverse Matrix	164

25. METHOD OF STEEPEST DESCENT. CONVERGENCE OF THE PROCEDURE

Idea of the Method	166
Convergence of the Procedure	168
Application to $ f(x+iy) ^2$	170

26. METHOD OF STEEPEST DESCENT. WEAKLY LINEAR CONVERGENCE OF THE ξ_μ

The Derived Set of the ξ_μ	173
Weakly Linear Convergence	174
Condition for the Regular Minimum of the Function (25.3)	176
Algebraic Equations with One Unknown	177

27. METHOD OF STEEPEST DESCENT. LINEAR CONVERGENCE OF THE ξ_μ

Conditions for Strictly Linear Convergence	178
An Example	180
Connection with the Newton–Raphson Procedure	183

28. CONVERGENT PROCEDURES FOR POLYNOMIAL EQUATIONS

The First Step of the Procedure	186
Convergence of the Iteration Procedure	188
Switching over to the Newton–Raphson Procedure	190
The Ω -Test	190

29. J -TEST AND J -ROUTINE

Basic Theorem	194
The J -Test	196
The J_m -Routine	197

30. q -ACCELERATION. THE PRACTICE OF THE PROCEDURE

The Definition of q -Acceleration	200
The Basic Lemma	201
The Convergence Discussion	203
Speed of Convergence	203
Flow Charts	205

31. NORMED LINEAR SPACES

Linear Spaces	207
Norms	208
Convergence	209
Completeness and Compactness	210
Examples	210
Spaces $C^k(J)$	211
Spaces $L_\alpha(G)$	212

32. METRIC SPACES

Definition of Metric Spaces	214
Principle of Contracting Operators	215

33. OPERATORS IN NORMED LINEAR SPACES

Mappings and Operators	219
Bounded Operators	219
Linear Operators	220
Strong and Weak Convergence	221

34. INVERSE OPERATORS

Definition of the Inverse Operator	224
Existence of the Inverse Operator	225
Another Existence Theorem	226
A Banach Theorem	227

35. OPERATORS MAPPING A LINEAR INTERVAL

A Refinement of Borel's Covering Theorem	229
Lipschitz Condition for $H(t)$	231
Taylor Development	233

36. THE DIRECTIONAL DERIVATIVES AND GRADIENTS OF OPERATORS

Directional Derivatives	236
Gateau Gradient	237
<i>F</i> -Differentials and <i>F</i> -Gradients	239

37. CENTRAL EXISTENCE THEOREM

Formulation of the Central Existence Theorem	241
A Local Existence Theorem	242
Proof of Theorem 37.1	245

38. NEWTON–RAPHSON ITERATION IN BANACH SPACES. STATEMENT OF THE THEOREMS

Definition of the α_v	247
Formulation of Theorems 38.1–38.3	248
A Lemma	250

39. PROOF OF THEOREMS 38.1–38.3

A Further Lemma	253
Specialization for Quadratic Polynomials	254
Proofs of Theorems 38.1–38.3	256

40. COMPLEMENTS TO THE NEWTON–RAPHSON METHOD

Equalities in Estimates for Quadratic Polynomials	260
Multiple Solutions	260
Unicity Theorem	261

41. CENTRAL EXISTENCE THEOREM FOR FINITE SYSTEMS OF EQUATIONS

Formulation of the Central Existence Theorem	265
The Choice of Norms	266
A Uniqueness Theorem	266
Example	267

42. NEWTON–RAPHSON ITERATION FOR FINITE SYSTEMS OF EQUATIONS

Formulation of the Theorem	270
The Choice of the Norms	271
Application to Complex Functions of a Complex Variable	274

APPENDICES

A. Continuity of the Roots of Algebraic Equations	276
B. Relative Continuity of the Roots of Algebraic Equations	281
C. An Explicit Formula for the n th Derivative of the Inverse Function	290
D. Analog of the Regula Falsi for Two Equations with Two Unknowns	294
E. Steffensen's Improved Iteration Rule	296
F. The Newton–Raphson Algorithm for Quadratic Polynomials	301
G. Some Modifications and Improvements of the Newton–Raphson Method	306
H. Rounding Off in Inverse Interpolation	310
I. Accelerating Iterations with Supralinear Convergence	320
J. Roots of $f(z) = 0$ in Terms of the Coefficients of the Development of $1/f(z)$	324
K. Continuity of the Fundamental Roots as Functions of the Elements of the Matrix	334
L. The Determinantal Formulas for Divided Differences	336
M. Remainder Terms in Interpolation Formulas	339
N. Generalization of Schröder's Series to the Case of Multiple Roots	343
O. Laguerre Iterations.	353
P. Approximation of Equations by Algebraic Equations of a Given Degree. Asymptotic Errors for Multiple Zeros	363
Q. Feedback Techniques for Error Estimates	372
A Numerical Example	377
R. Reduced Polynomial Equations	378
S. Discussion of the q -Acceleration	383
T. Remainder in the Taylor Formula for Analytic Functions	389
U. Equality Conditions for the Newton–Raphson Iteration	391
A Lemma	391
Equality Conditions for Normed Spaces	392
Equality Conditions for Strictly Normed Spaces	394
 Bibliographical Notes	 399
 INDEX	 409

