

## **Contents**

1	INTF	RODUCTION	1	
	1.1	Dimensional Analysis, 1		
	1.2	Expansions, 10		
	1.3	Gauge Functions, 12		
	1.4	Order Symbols, 17		
	1.5	Asymptotic Series, 18		
	1.6	Asymptotic Expansions and Sequences, 22		
	1.7	Convergent Versus Asymptotic Series, 23		
	1.8	Elementary Operations on Asymptotic Expansions, 24		
		Exercises, 24		
2	ALGEBRAIC EQUATIONS			
	2.1	Quadratic Equations, 28		
	2.2	Cubic Equations, 39		
	2.3	Higher-Order Equations, 43		
	2.4	Transcendental Equations, 45		
		Exercises, 48		
3	INTE	GRALS	51	
	3.1	Expansion of Integrands, 52		
	3.2	Integration by Parts, 56		
	3.3	Laplace's Method, 65		
	3.4	The Method of Stationary Phase, 79		

v	CO	NIT	EN	T	C
Х	CO	<b>TA T</b>	Er	4 T	N

	3.5	The Method of Steepest Descent, 88	
		Exercises, 101	
4	THE	DUFFING EQUATION	107
	4.1	The Straightforward Expansion, 109	
	4.2	Exact Solution, 113	
	4.3	The Lindstedt-Poincaré Technique, 118	
	4.4	The Method of Renormalization, 121	
	4.5	The Method of Multiple Scales, 122	
	4.6	Variation of Parameters, 127	
	4.7	The Method of Averaging, 129	
		Exercises, 131	
5	THE	LINEAR DAMPED OSCILLATOR	134
	5.1	The Straightforward Expansion, 135	
	5.2	Exact Solution, 136	
	5.3	The Lindstedt-Poincaré Technique, 139	
	5.4	The Method of Multiple Scales, 142	
	5.5	The Method of Averaging, 144	
		Exercises, 146	
6	SELI	F-EXCITED OSCILLATORS	147
	6.1	The Straightforward Expansion, 148	
	6.2	The Method of Renormalization, 151	
	6.3	The Method of Multiple Scales, 152	
	6.4	The Method of Averaging, 155	
		Exercises, 157	
7	SYS	TEMS WITH QUADRATIC AND CUBIC NONLINEARITIES	159
	7 1	The Straightforward Expansion 160	

	7.2	The Method of Renormalization, 162	
	7.3	The Lindstedt-Poincaré Technique, 164	
	7.4	The Method of Multiple Scales, 166	
	7.5	The Method of Averaging, 168	
	7.6	The Generalized Method of Averaging, 169	
	7.7	The Krylov-Bogoliubov-Mitropolsky Technique, 173	
		Exercises, 175	
8	GEN	ERAL WEAKLY NONLINEAR SYSTEMS	177
	8.1	The Straightforward Expansion, 177	
	8.2	The Method of Renormalization, 179	
	8.3	The Method of Multiple Scales, 181	
	8.4	The Method of Averaging, 182	
	8.5	Applications, 184	
		Exercises, 188	
9	FOR	CED OSCILLATIONS OF THE DUFFING EQUATION	190
	9.1	The Straightforward Expansion, 191	
	9.2	The Method of Multiple Scales, 193	
		9.2.1 Secondary Resonances, 193	
		9.2.2 Primary Resonance, 205	
	9.3	The Method of Averaging, 209	
		9.3.1 Secondary Resonances, 209	
	_	9.3.2 Primary Resonance, 212	
	Exe	rcises, 213	
10	MUL	TIFREQUENCY EXCITATIONS	216
	10.1	The Straightforward Expansion, 216	4
	10.2	The Method of Multiple Scales, 219	
		10.2.1 The Case $\omega_2 + \omega_1 \approx 1,220$	

:	:	~	<b>~</b> >	TT	וכדי	NIT	2
X1	1	u	H	VІ	E.	NI	3

		10.3.1 The Case $\omega_1 + \omega_2 \approx 1,230$	
		10.3.2 The Case $\omega_2 - \omega_1 \approx 1$ and $\omega_1 \approx 2,230$	
		Exercises, 230	
11	ТНЕ	MATHIEU EQUATION	234
	11.1	The Straightforward Expansion, 235	
	11.2	The Floquet Theory, 236	
	11.3	The Method of Strained Parameters, 243	
	11.4	Whittaker's Method, 247	
	11.5	The Method of Multiple Scales, 249	
	11.6	The Method of Averaging, 253	
		Exercises, 254	
12	BOU	NDARY-LAYER PROBLEMS	257
	12.1	A Simple Example, 257	
	12.2	The Method of Multiple Scales, 268	
	12.3	The Method of Matched Asymptotic Expansions, 270	
	12.4	Higher Approximations, 279	
	12.5	Equations with Variable Coefficients, 284	
	12.6	Problems with Two Boundary Layers, 296	
	12.7	Multiple Decks, 304	
	12.8	Nonlinear Problems, 307	
		Exercises, 320	
13	LINE	EAR EQUATIONS WITH VARIABLE COEFFICIENTS	325
	13.1	First-Order Scalar Equations, 326	
	13.2	Second-Order Equations, 329	
	13.3	Solutions Near Regular Singular Points, 331	

10.2.2 The Case  $\omega_2$  -  $\omega_1 \approx 1$  and  $\omega_1 \approx 2$ , 222

10.3 The Method of Averaging, 226

13.5	Solutions Near an Irregular Singular Point, 344  Exercises, 355	
DIFF	ERENTIAL EQUATIONS WITH A LARGE PARAMETER 3	60
14.1	The WKB Approximation, 361	
14.2	The Liouville-Green Transformation, 364	
14.3	Eigenvalue Problems, 366	
14.4	Equations with Slowly Varying Coefficients, 369	
14.5	Turning-Point Problems, 370	
14.6	The Langer Transformation, 375	
14.7	Eigenvalue Problems with Turning Points, 379	
	Exercises, 383	
SOL	VABILITY CONDITIONS 3	888
15.1	Algebraic Equations, 389	
15.2	Nonlinear Vibrations of Two-Degree-of-Freedom Gyroscopic Systems, 394	
15.3	Parametrically Excited Gyroscopic Systems, 397	
15.4	Second-Order Differential Systems, 401	
15.5	General Boundary Conditions, 406	
15.6	A Simple Eigenvalue Problem, 412	
15.7	A Degenerate Eigenvalue Problem, 414	
15.8	Acoustic Waves in a Duct with Sinusoidal Walls, 418	
15.9	Vibrations of Nearly Circular Membranes, 426	
15.10	A Fourth-Order Differential System, 432	
15.1	General Fourth-Order Differential Systems, 438	
15.12	2 A Fourth-Order Eigenvalue Problem, 441	
15.13	3 A Differential System of Equations, 445	
15.14	4 General Differential Systems of First-Order Equations, 447	
15 15	5 Differential Systems with Interfacial Boundary Conditions, 452	

13.4 Singularity at Infinity, 342

14

15

## xiv CONTENTS

15.16 Integral Equations, 454

15.17 Partial-Differential Equations, 458

Exercis	ses, 462	
APPENDIX A	TRIGONOMETRIC IDENTITIES	472
APPENDIX B EQUATIONS	LINEAR ORDINARY-DIFFERENTIAL	480
BIBLIOGRAPI	нү	501
INDEX		507

