
Contents

<i>Preface</i>	v
1: Classification of integral equations	
1.1 Historical introduction	1
1.2 Linear integral equations	3
1.3 Special types of kernel	4
1.3.1 Symmetric kernels	4
1.3.2 Kernels producing convolution integrals	5
1.3.3 Separable kernels	6
1.4 Square integrable functions and kernels	8
1.5 Singular integral equations	9
1.6 Non-linear equations	11
Problems	12
2: Connection with differential equations	
2.1 Linear differential equations	14
2.2 Green's function	18
2.3 Influence function	20
Problems	22
3: Integral equations of the convolution type	
3.1 Integral transforms	24
3.2 Fredholm equation of the second kind	26
3.3 Volterra equation of the second kind	31
3.4 Fredholm equation of the first kind	34
3.4.1 Stieltjes integral equation	34
3.5 Volterra equation of the first kind	36
3.5.1 Abel's integral equation	37
3.6 Fox's integral equation	39
Problems	40
4: Method of successive approximations	
4.1 Neumann series	43
4.2 Iterates and the resolvent kernel	46
Problems	51

5: Integral equations with singular kernels

5.1	Generalization to higher dimensions	53
5.2	Green's functions in two and three dimensions	54
5.3	Dirichlet's problem	55
5.3.1	Poisson's formula for the unit disc	59
5.3.2	Poisson's formula for the half plane	60
5.3.3	Hilbert kernel	61
5.3.4	Hilbert transforms	63
5.4	Singular integral equation of Hilbert type	65
	Problems	67

6: Hilbert space

6.1	Euclidean space	69
6.2	Hilbert space of sequences	71
6.3	Function space	74
6.3.1	Orthonormal system of functions	75
6.3.2	Gram-Schmidt orthogonalization	76
6.3.3	Mean square convergence	77
6.3.4	Riesz-Fischer theorem	79
6.4	Abstract Hilbert space	80
6.4.1	Dimension of Hilbert space	82
6.4.2	Complete orthonormal system	82
	Problems	83

7: Linear operators in Hilbert space

7.1	Linear integral operators	85
7.1.1	Norm of an integral operator	87
7.1.2	Hermitian adjoint	88
7.2	Bounded linear operators	89
7.2.1	Matrix representation	91
7.3	Completely continuous operators	92
7.3.1	Integral operator with square integrable kernel	93
	Problems	95

8: The resolvent

8.1	Resolvent equation	98
8.2	Uniqueness theorem	99
8.3	Characteristic values and functions	101

8.4	Neumann series	102
8.4.1	Volterra integral equation of the second kind	105
8.4.2	Bôcher's example	109
8.5	Fredholm equation in abstract Hilbert space	109
	Problems	111

9: Fredholm theory

9.1	Degenerate kernels	114
9.2	Approximation by degenerate kernels	120
9.3	Fredholm theorems	121
9.3.1	Fredholm theorems for completely continuous operators	125
9.4	Fredholm formulae for continuous kernels	126
	Problems	135

10: Hilbert-Schmidt theory

10.1	Hermitian kernels	136
10.2	Spectrum of a Hilbert-Schmidt kernel	136
10.3	Expansion theorems	139
10.3.1	Hilbert-Schmidt theorem	141
10.3.2	Hilbert's formula	143
10.3.3	Expansion theorem for iterated kernels	143
10.4	Solution of Fredholm equation of second kind	144
10.5	Bounds on characteristic values	146
10.6	Positive kernels	147
10.7	Mercer's theorem	148
10.8	Variational principles	150
10.8.1	Rayleigh-Ritz variational method.	152
	Problems	154

<i>Bibliography</i>	157
<i>Index</i>	158