CONTENTS

INTRODUCTION 1

CHAPTER I NEW CLASS OF SINGULAR INTEGRAL EQUATIONS 13

1. Certain Classes of Functions 13
1.1. Notation 13
1.2. Classes of Functions with Isolated Subsets 14
1.3. Classes of Functions with Singularities 16

2. Properties of Some Integrals 19
2.1. Investigation of the Integral 19
2.2. On some Properties of the Integral 24
2.3. Properties of the Simplest Singular Function $\omega(x)$. 27
2.4. Case of a Singular Boundary Point 29
2.5. Generalization to an Infinite Domain 32
2.6. Case of Several Singular Points 33

3. Integrals with Singular Manifolds 35
3.1. Singular Manifolds 35
3.2. Three Lemmas 36
3.3. Investigation of the Simplest Singular Function $\omega_m(x)$ 39
3.4. Case of a Boundary Singular Manifold 43
3.5. Generalization to Unbounded Domains 45
3.6. Case of Several Manifolds 45

4. Investigation of the Simplest Operator 46
4.1. Fundamental Theorem 46
4.2. Linearity of the Operator in Spaces of Type C and H 49
4.3. Incomplete Continuity of the Operator 55

5. General Operator 59
5.1. Linearity of the Operator in $S(\beta, D)$ 59
5.2. Theorem on Complete Continuity 61

5.3. Operator $K\varphi$ in C and H Type Spaces 64
5.4. On the Unimproveability of Theorem 5.3 66

6. Integral Equations 66
6.1. Existence and Uniqueness 66
6.2. Fredholm Theorems 67
6.3. Most General and Simplest Equations 71
6.4. Volterra Type Equations 73
6.5. Equation in the Whole Space 78

CHAPTER II ELLIPTIC EQUATIONS WITH SINGULAR COEFFICIENTS 81
1. Classification. Transformation 81
1.1. Classification of Singular Points 81
1.2. Transformation of the Equation with Inversion . . 82

2. Generalized Maximum Principle and Some Properties of the Solutions of Boundary Value Problems . 86
2.1. Maximum Principle 86
2.2. Realization of Condition (2.2) 87
2.3. Formulation of Boundary Value Problems and Uniqueness 89

3. Manifold of Solutions 90
3.1. Local Investigation of a Regular Singular Point . . 90
3.2. Generalization 96
3.3. Equations in all Space 99

4. Boundary Value Problems 102
4.1. Reduction to an Integral Equation and Fundamental Theorems 102
4.2. Generalization to Singular Manifolds 108
4.3. First Exterior Boundary Value Problem 111
4.4. Another Method of Reducing Boundary Value Problems 112
4.5. On the Behavior of the Solution at a Singular Point 113
4.6. The Special Case $n = 2$, $c(0) \neq 0$ 115
4.7. Example 118

CHAPTER III GENERALIZED CAUCHY-RIEMANN SYSTEM WITH SINGULAR COEFFICIENTS 121
1. Investigation of the Fundamental Integral and Other Preliminary Information 121
1.1. Complex Differentiation and Integration 121
1.2. Extension of Complex Differentiation and Integration to a Function with Isolated Singularities . . 124
1.3. Investigation of the Fundamental Integral 127
1.4. Examples (see [42], [43], [46]) 131
2. Fundamental Theorems 134
2.1. On the Behavior of the Solutions at the Singular Point 134
2.2. Violation of the Liouville Theorem. 136
2.3. Fundamental Theorems 138
2.4. Inhomogeneous Equation 142
2.5. Extension to an Unbounded Domain 143
2.6. Equations with p Singular Points 144
2.7. Equations with Singular Lines 146
3. Boundary Value Problems 149
3.1. Formulation of the Problems and their Qualitative Investigation 149
3.2. Exact Theorem on Solvability. 151
3.3. Particular Case 152
3.4. Problem with a Skew Derivative for a Second Order Equation 153
3.5. Application to Problems of the Flexure of Non-regular 155
4. Explicitly Solvable Integral Equations 158
4.1. Homogeneous Equation 159
4.2. Extraction of Summable Solutions 163
4.3. The Transpose Equation 165
4.4. Inhomogeneous Equation 167
4.5. Examples 170

CHAPTER IV GENERAL BOUNDARY VALUE PROBLEM OF THE LINEAR CONJUGATE 175
1. Direct Study of the Problem (A) in the Elliptic Case 176

1.1. Fundamental Theorem 176
1.2. The Problem (A) in the Case of Open Contours . . 179
1.3. Nonlinear Boundary Value Problems 183
1.4. Boundary Value Problems with a Derivative . . . 185
1.5. Qualitative Investigation of the Homogeneous Problem 186

2. Parabolic Case 188
2.1. Auxiliary Problem 188
2.2. Investigation of Problem (A) 190
2.3. Problem with Translation (Shift) 193
2.4. Generalization of the Boundary Condition 194

3. On a Class of Singular Integral Equations 195

4. Relation of the Boundary Value Problem (A) To Integral Equations 198
4.1. Noether Theorem for Boundary Value Problem (A) and the Fundamental Theorem 198
4.2. Problem (A) with Translation 201

5. Exact Theorems on the Solvability of Some Integral Equations 205
5.1. Generalization of the Characteristic Singular Integral Equation 205
5.2. Integral Equations of Plane Potential Theory . . . 206

6. Conjugate Problem of Generalized Analytic Funktions 210

7. Conjugate Problem of the Solutions of a Second Order Differential Equation 216

CITED LITERATURE 220