



# CONTENTS

<i>Preface</i>	vii
<i>Contents of Volume II</i>	xv
<i>Introduction</i>	xvii

## I. Asymptotic Expansions

1.1. The Order Symbols $O$ and $o$	1
1.2. Definition of an Asymptotic Expansion	2
1.3. Elementary Properties of Asymptotic Series	3
1.4. Watson's Lemma	4

## II. The Gamma Function and Related Functions

2.1. Definitions and Elementary Properties	8
2.2. Analytic Continuation of $\Gamma(z)$	10
2.3. Multiplication Formula	11
2.4. The Logarithmic Derivative of the Gamma Function	12
2.5. Integral Representations for $\psi(z)$ and $\ln \Gamma(z)$	13
2.6. The Beta Function and Related Functions	15
2.7. Contour Integral Representations for Gamma and Beta Functions	16
2.8. Bernoulli Polynomials and Numbers	18
2.9. The $D$ and $\delta$ Operators	24
2.10. Power Series and Other Expansions	26
2.11. Asymptotic Expansions	31

## III. Hypergeometric Functions

3.1. Elementary Hypergeometric Series	38
3.2. A Generalization of the ${}_2F_1$	41
3.3. Convergence of the ${}_pF_q$ Series	43
3.4. Elementary Relations	44
3.5. The Confluence Principle	48
3.6. Integral Representations	57
3.7. Differential Equations for the ${}_2F_1$	64
3.8. Kummer's Solutions	67
3.9. Analytic Continuation	69

3.10. The Complete Solution	72
3.11. Kummer-Type Relations for the Logarithmic Solutions	85
3.12. Quadratic Transformations	92
3.13. The ${}_{p+1}F_p$ for Special Values of the Argument	99

#### IV. Confluent Hypergeometric Functions

4.1. Introduction	115
4.2. Integral Representations	115
4.3. Elementary Relations for the Confluent Functions	117
4.4. Confluent Differential Equation	119
4.5. The Complete Solution	121
4.6. Kummer-Type Relations for the Logarithmic Solutions	124
4.7. Asymptotic Expansions for Large $z$	127
4.8. Asymptotic Behavior for Large Parameters and Variable	129
4.9. Other Notations and Related Functions	134

#### V. The Generalized Hypergeometric Function and the G-Function

5.1. The ${}_pF_q$ Differential Equation	136
5.2. The G-Function	143
5.3. Analytic Continuation of $G_{p,p}^{m,n}(z)$	148
5.4. Elementary Properties of the G-Function	149
5.5. Multiplication Theorems	152
5.6. Integrals Involving G-Functions	157
5.7. Asymptotic Expansion of $G_{p,q}^{q,1}(z)$ and $G_{p,q}^{q,0}(z)$ for Large $z$	178
5.8. Differential Equation for $G_{p,q}^{m,n}(z)$	181
5.9. Series of G-Functions	183
5.10. Asymptotic Expansions of $G_{p,q}^{m,n}(z)$	189
5.11. Asymptotic Expansions of ${}_pF_q(z)$ for Large $z$	195

#### VI. Identification of the ${}_pF_q$ and G-Functions with the Special Functions of Mathematical Physics

6.1. Introduction	209
6.2. Named Special Functions Expressed as ${}_pF_q$ 's	209
6.3. The ${}_pF_q$ Expressed as a Named Function	224
6.4. Named Functions Expressed in Terms of the G-Function	225
6.5. The G-Function Expressed as a Named Function	230

#### VII. Asymptotic Expansions of ${}_pF_q$ for Large Parameters

7.1. Introduction	235
7.2. The ${}_2F_1$	235
7.3. Some Generalizations of the ${}_2F_1$ Formulas	242
7.4. Extended Jacobi Polynomials	247

**VIII. Orthogonal Polynomials**

8.1. Orthogonal Properties	267
8.2. Jacobi Polynomials	274
8.3. Expansion of Functions in Series of Jacobi Polynomials	283
8.4. Evaluation and Estimation of the Coefficients in the Expansion of a Given Function $f(x)$ in Series of Jacobi Polynomials	286
8.5. Chebyshev Polynomials	296
8.6. Differential and Integral Properties of Expansions in Series of Chebyshev Polynomials of the First Kind	314
8.7. A Nesting Procedure for the Computation of Expansions in Series of Functions Where the Functions Satisfy Linear Finite Difference Equations	325

<b>Bibliography</b>	330
---------------------	-----

<i>Notation Index</i>	339
-----------------------	-----

<i>Subject Index</i>	344
----------------------	-----