



## Contents

<i>Preface</i> . . . . .	ix
<b>Chapter 1. LINEAR DIFFERENTIAL OPERATORS WITH ALMOST PERIODIC COEFFICIENTS</b> . . . . .	<b>1</b>
§1. Almost Periodic Functions . . . . .	1
1.1. Spaces of ap-functions (1). 1.2. Approximation theorem (3). 1.3. Differentiation and integration of ap-functions (4). 1.4. The superposition operator (5).	
§2. Regular AP-Operators . . . . .	5
2.1. Statement of the problem (5). 2.2. Esclangon's theorem (8). 2.3. Existence of bounded solutions when the right-hand sides are bounded (11). 2.4. Uniqueness theorem (13). 2.5. Equivalence theorem (16).	
§3. Behavior of Solutions of the Homogeneous Equation . . . . .	19
3.1. First dichotomy theorem (19). 3.2. Exponential dichotomy (20). 3.3. Uniformity of dichotomy (23).	
§4. The Green's Function. . . . .	25
4.1. Description of the Green's function (25). 4.2. Construction of the Green's function (26). 4.3. Dependence of the Green's function on a parameter (28). 4.4. Examples (32). 4.5. The Green's function of a periodic boundary-value problem (34).	
§5. Positivity of the Green's Function . . . . .	35
5.1. Wedges and cones (35). 5.2. Functionals selecting cones (36). 5.3. Space of cones (39). 5.4. The cones $\hat{K}(t)$ (41). 5.5. Functionals selecting $K(t)$ (44). 5.6. Positive linear operators (46). 5.7. Green's functions of constant sign (48). 5.8. Converses of theorems on positivity of an integral operator (51).	
<b>Chapter 2. ANALYSIS OF SPECIFIC TYPES OF AP-OPERATORS</b> . . . . .	<b>53</b>
§6. First-Order Systems of Equations . . . . .	53
6.1. General positivity conditions for the Green's function (53). 6.2. Gâteaux derivatives (55). 6.3. Positivity of the translation operator (59). 6.4. Positivity of the translation operator with respect to a faceted cone (64). 6.5. On construction of cone-valued functions $K(t)$ (66). 6.6. On the stability of solutions of the homogeneous equation (68).	
§7. Second-Order Systems . . . . .	71
7.1. Equations with constant coefficients (71). 7.2. The majorant test (73). 7.3. Use of first-order systems (75). 7.4. Strong positivity (82). 7.5. More about systems with constant coefficients (87). 7.6. Remark on first-order systems (89).	

§8. Higher-Order Scalar Equations . . . . .	89
8.1. Statement of the problem (89). 8.2. A necessary condition for the Green's function of the ap-operator (8.10) to be of constant sign (92). 8.3. Sufficient conditions for the Green's function of the ap-operator (8.10) to be of constant sign (95). 8.4. Special classes of operators with constant coefficients (102). 8.5. Operators with variable coefficients (103).	
§9. Second-Order Scalar AP-Operators . . . . .	109
9.1. Connection with oscillatory properties of solutions of the homogeneous equation (109). 9.2. The Riccati equation (111). 9.3. Differential inequalities for the Riccati equation (114). 9.4. Almost periodic solutions of the Riccati equation (118). 9.5. Stability of solutions of equations (9.2) (121). 9.6. Necessary and sufficient conditions for existence and constancy of sign of the Green's function (124). 9.7. Completion of proof of Theorem 9.1 (126). 9.8. Integral test for nonoscillation (126).	
<i>Chapter 3. GLOBAL THEOREMS ON AP-SOLUTIONS OF</i>	
<i>NONLINEAR EQUATIONS . . . . .</i>	<i>129</i>
§10. General Existence Theorems . . . . .	129
10.1. Passage to the integral equation (1290). 10.2. Generalized contracting-mapping principle (132). 10.3. Equations with monotone operators (133). 10.4. Uniformly concave cones (135). 10.5. Equations with monotone nonlinearities (137). 10.6. Equations with uniformly concave nonlinearities (139). 10.7. Use of majorants and minorants (141). 10.8. Examples (144).	
§11. Stability of AP-Solutions of Equations with Monotone and Concave Nonlinearities . . . . .	149
11.1. Stability of the trivial solution (149). 11.2. Instability theorem (152). 11.3. Stability of the operator L and the sign of its Green's function (153). 11.4. Stability of ap-solutions of equations with monotone nonlinearities (154). 11.5. Lemma on concave operators (156). 11.6. Stability of ap-solutions of equations with concave nonlinearities (157). 11.7. Stability in the cone of first-order systems with concave nonlinearities (159). 11.8. Stability in the cone of solutions of higher-order systems with concave nonlinearities (167). 11.9. Stability of ap-solution under small perturbations of the right-hand side of the equation (173). 11.10. Examples (174).	
§12. Almost Periodic Oscillations in Automatic Control Systems . . . . .	174
12.1. Introductory remarks (174). 12.2. Undisturbed automatic control system (175). 12.3. Existence of an ap-solution of the disturbed system (183). 12.4. Existence of an unstable ap-solution of the disturbed system (187). 12.5. Applications to scalar equations of higher order (195). 12.6. Existence of three ap-solutions (200).	
§13. Positive Almost-Periodic Solutions of Second-Order Equations . . . . .	206
13.1. Statement of results (206). 13.2. Proof of Theorem 13.1 (209). 13.3. Lemma on positive solvability of the differential inequality (210). 13.4. Proof of Theorem 13.2 (213). 13.5. Proof of Theorem 13.3 (215). 13.6. Regularity of auxiliary ap-operators (219). 13.7. Proof of Theorem 13.4 (222).	
<i>Chapter 4. EQUATIONS WITH A SMALL PARAMETER . . . . .</i>	<i>226</i>
§14. Linear Equations . . . . .	226
14.1. Bogolyubov's lemma (226). 14.2. Dependence of the Green's function on a parameter (228). 14.3. The Bogolyubov-Shtokalo substitutions (232). 14.4. Formulas of perturbation theory (233). 14.5. Shtokalo's theorem (235).	

§ 15. Bifurcation of Almost-Periodic Solutions . . . . .	238
15.1. Statement of the problem (238). 15.2. Passage to an equation of special form (240). 15.3. Fundamental theorem (242). 15.4. Proof of the existence of ap-solutions (243). 15.5. Invariant cones (249). 15.6. Uniqueness of ap-solution in cone (253). 15.7. End of the proof of Theorem 15.1 (256). 15.8. Pendulum with vibrating point of suspension (258). 15.9. Further remarks (260).	
§ 16. Bifurcation of AP-Solutions of Singularly Perturbed Second-Order Equations . . . . .	260
16.1. Statement of the problem. Necessary condition (260). 16.2. Fundamental theorem (262). 16.3. Proof of part 1 (263). 16.4. Existence of nontrivial ap-solutions in parts 2 and 3 (264). 16.5. End of the proof of Theorem 16.1 (266). 16.6. Functions $a(t)$ of constant sign (269). 16.7. General equation (269). 16.8. Example (271).	
<i>Summary of Chapters 1–4.</i> . . . . .	273
§ 1. Almost periodic functions (273). § 2. Regular ap-operators (274). § 3. Behavior of solutions of the homogeneous equation (276). § 4. The Green's function (277). § 5. Positivity of the Green's function (279). § 6. First-order systems of equations (282). § 7. Second-order systems (286). § 8. Higher-order scalar equations (289). § 9. Second-order scalar ap-operators (295). § 10. General existence theorems (297). § 11. Stability of ap-solutions of equations with monotone and concave nonlinearities (301). § 12. Almost periodic oscillations in automatic control systems (304). § 13. Positive almost-periodic solutions of second-order equations (308). § 14. Linear equations (311). § 15. Bifurcation of almost-periodic solutions (314). § 16. Bifurcation of ap-solutions of singularly perturbed second-order equations (317).	
<i>Bibliography</i> . . . . .	319
<i>Subject Index</i> . . . . .	325