

Contents

Preface	vii
General notation	xiv
Chapter I. Preliminaries	1
1. Basic notions and notations, 1	
2. Probability measures on a metric space, 2	
3. Expectations, conditional expectations and regular conditional probabilities, 11	
4. Continuous stochastic processes, 16	
5. Stochastic processes adapted to an increasing family of sub σ -fields, 20	
6. Martingales, 25	
7. Brownian motions, 40	
8. Poisson random measures, 42	
9. Point processes and Poisson point processes, 43	
Chapter II. Stochastic integrals and Itô's formula	45
1. Itô's definition of stochastic integrals, 45	
2. Stochastic integrals with respect to martingales, 53	
3. Stochastic integrals with respect to point processes, 59	
4. Semi-martingales, 63	
5. Itô's formula, 66	
6. Martingale characterization of Brownian motions and Poisson point processes, 73	
7. Representation theorem for semi-martingales, 84	
Chapter III. Stochastic calculus	97
1. The space of stochastic differentials, 97	
2. Stochastic differential equations with respect to quasimartingales, 103	
3. Moment inequalities for martingales, 110	
4. Some applications of stochastic calculus to Brownian motions, 113	
4.1. Brownian local time	
4.2. Reflecting Brownian motion and the Skorohod equation	
4.3. Excursions of Brownian motion	

xii CONTENTS

- 4.4. Some limit theorems for occupation times of Brownian motion
- 5. Exponential martingales, 140

Chapter IV. Stochastic differential equations145

- 1. Definition of solutions, 145
- 2. Existence theorem, 153
- 3. Uniqueness theorem, 164
- 4. Solution by transformation of drift and by time change, 176
- 5. Diffusion processes, 188
- 6. Diffusion processes generated by differential operators and stochastic differential equations, 198
- 7. Stochastic differential equations with boundary conditions, 203
- 8. Examples, 218
- 9. Stochastic differential equations with respect to Poisson point processes, 230

Chapter V. Diffusion processes on manifolds233

- 1. Stochastic differential equations on manifolds, 233
- 2. Flow of diffeomorphisms, 239
- 3. Heat equation on a manifold, 254
- 4. Non-degenerate diffusions on a manifold and their horizontal lifts, 260
- 5. Stochastic parallel displacement and heat equation for tensor fields, 282
- 6. The case with boundary conditions, 289
- 7. Malliavin's stochastic calculus of variation for Wiener functionals, 322
- 8. The case of stochastic differential equations and hypoellipticity problem of heat equations, 334

Chapter VI. Theorems on comparison and approximation and their applications352

- 1. A comparison theorem for one-dimensional Itô processes, 352
- 2. An application to an optimal control problem, 356
- 3. Some results on one-dimensional diffusion processes, 361
- 4. Comparison theorem for one-dimensional projection of diffusion processes, 367
- 5. Applications to diffusions on Riemannian manifolds, 375
- 6. Stochastic line integrals along the paths of diffusion processes, 382
- 7. Approximation theorems for stochastic integrals and stochastic differential equations, 392
- 8. The support of diffusion processes, 429
- 9. Asymptotic evaluation of the diffusion measure for tubes around a smooth curve, 444

Bibliography453

Index461

