Contents

Preface xi

1

Ordinary Differential Equations 1

1.1 Introduction *1*
1.2 Linear Dependence *3*
1.3 Complete Solutions of Linear Equations *4*
1.4 The Linear Differential Equation of First Order *6*
1.5 Linear Differential Equations with Constant Coefficients *8*
1.6 The Equidimensional Linear Differential Equation *12*
1.7 Properties of Linear Operators *15*
1.8 Simultaneous Linear Differential Equations *18*
1.9 Particular Solutions by Variation of Parameters *24*
1.10 Reduction of Order *28*
1.11 Determination of Constants *30*
1.12 Special Solvable Types of Nonlinear Equations *31*

2

The Laplace Transform 53

2.1 An introductory Example *53*
2.2 Definition and Existence of Laplace Transforms *55*

2.3 Properties of Laplace Transforms *58*
2.4 The Inverse Transform *62*
2.5 The Convolution *63*
2.6 Singularity Functions *65*
2.7 Use of Table of Transforms *67*
2.8 Applications to Linear Differential Equations with Constant Coefficients *72*
2.9 The Gamma Function *76*

3

Numerical Methods for Solving Ordinary Differential Equations **93**

3.1 Introduction *93*
3.2 Use of Taylor Series *94*
3.3 The Adams Method *96*
3.4 The Modified Adams Method *100*
3.5 The Runge–Kutta Method *102*
3.6 Picard's Method *105*
3.7 Extrapolation with Differences *107*

4

Series Solutions of Differential Equations: Special Functions **118**

4.1 Properties of Power Series *118*
4.2 Illustrative Examples *122*
4.3 Singular Points of Linear Second-Order Differential Equations *126*
4.4 The Method of Frobenius *128*
4.5 Treatment of Exceptional Cases *134*
4.6 Example of an Exceptional Case *136*
4.7 A Particular Class of Equations *138*
4.8 Bessel Functions *141*
4.9 Properties of Bessel Functions *147*
4.10 Differential Equations Satisfied by Bessel Functions *151*
4.11 Ber and Bei Functions *153*
4.12 Legendre Functions *155*
4.13 The Hypergeometric Function *162*
4.14 Series Solutions Valid for Large Values of x *163*

5

Boundary-Value Problems and Characteristic-Function Representations 186

5.1 Introduction *186*
5.2 The Rotating String *188*
5.3 The Rotating Shaft *192*
5.4 Buckling of Long Columns Under Axial Loads *195*
5.5 The Method of Stodola and Vianello *197*
5.6 Orthogonality of Characteristic Functions *203*
5.7 Expansion of Arbitrary Functions in Series of Orthogonal Functions *207*
5.8 Boundary-Value Problems Involving Nonhomogeneous Differential Equations *211*
5.9 Convergence of the Method of Stodola and Vianello *212*
5.10 Fourier Sine Series and Cosine Series *214*
5.11 Complete Fourier Series *219*
5.12 Term-by-Term Differentiation of Fourier Series *223*
5.13 Fourier–Bessel Series *226*
5.14 Legendre Series *230*
5.15 The Fourier Integral *234*

6

Vector Analysis 269

6.1 Elementary Properties of Vectors *269*
6.2 The Scalar Product of Two Vectors *271*
6.3 The Vector Product of Two Vectors *273*
6.4 Multiple Products *275*
6.5 Differentiation of Vectors *277*
6.6 Geometry of a Space Curve *278*
6.7 The Gradient Vector *281*
6.8 The Vector Operator ∇ *283*
6.9 Differentiation Formulas *284*
6.10 Line Integrals *287*
6.11 The Potential Function *291*
6.12 Surface Integrals *294*
6.13 Interpretation of Divergence. The Divergence Theorem *297*
6.14 Green's Theorem *301*
6.15 Interpretation of Curl. Laplace's Equation *302*
6.16 Stokes's Theorem *303*
6.17 Orthogonal Curvilinear Coordinates *306*

6.18 Special Coordinate Systems *311*
6.19 Application to Two-Dimensional Incompressible Fluid Flow *313*
6.20 Compressible Ideal Fluid Flow *316*

7

Topics in Higher-Dimensional Calculus **342**

7.1 Partial Differentiation. Chain Rules *342*
7.2 Implicit Functions. Jacobian Determinants *347*
7.3 Functional Dependence *350*
7.4 Jacobians and Curvilinear Coordinates. Change of Variables in Integrals *352*
7.5 Taylor Series *354*
7.6 Maxima and Minima *356*
7.7 Constraints and Lagrange Multipliers *357*
7.8 Calculus of Variations *360*
7.9 Differentiation of Integrals Involving a Parameter *364*
7.10 Newton's Iterative Method *367*

8

Partial Differential Equations **384**

8.1 Definitions and Examples *384*
8.2 The Quasi-Linear Equation of First Order *387*
8.3 Special Devices. Initial Conditions *392*
8.4 Linear and Quasi-Linear Equations of Second Order *396*
8.5 Special Linear Equations of Second Order, with Constant Coefficients *397*
8.6 Other Linear Equations *400*
8.7 Characteristics of Linear First-Order Equations *403*
8.8 Characteristics of Linear Second-Order Equations *408*
8.9 Singular Curves on Integral Surfaces *414*
8.10 Remarks on Linear Second-Order Initial-Value Problems *417*
8.11 The Characteristics of a Particular Quasi-Linear Problem *417*

9

Solutions of Partial Differential Equations of Mathematical Physics **439**

9.1 Introduction *439*
9.2 Heat Flow *441*
9.3 Steady-State Temperature Distribution in a Rectangular Plate *443*
9.4 Steady-State Temperature Distribution in a Circular Annulus *446*

9.5 Poisson's Integral *450*
9.6 Axisymmetrical Temperature Distribution in a Solid Sphere *451*
9.7 Temperature Distribution in a Rectangular Parallelepiped *453*
9.8 Ideal Fluid Flow about a Sphere *456*
9.9 The Wave Equation. Vibration of a Circular Membrane *459*
9.10 The Heat-Flow Equation. Heat Flow in a Rod *461*
9.11 Duhamel's Superposition Integral *463*
9.12 Traveling Waves *467*
9.13 The Pulsating Cylinder *470*
9.14 Examples of the Use of Fourier Integrals *473*
9.15 Laplace Transform Methods *477*
9.16 Application of the Laplace Transform to the Telegraph Equations for a Long Line *480*
9.17 Nonhomogeneous Conditions. The Method of Variation of Parameters *484*
9.18 Formulation of Problems *490*
9.19 Supersonic Flow of Ideal Compressible Fluid Past an Obstacle *495*

10

Functions of a Complex Variable **539**

10.1 Introduction. The Complex Variable *539*
10.2 Elementary Functions of a Complex Variable *541*
10.3 Other Elementary Functions *544*
10.4 Analytic Functions of a Complex Variable *550*
10.5 Line Integrals of Complex Functions *554*
10.6 Cauchy's Integral Formula *560*
10.7 Taylor Series *561*
10.8 Laurent Series *563*
10.9 Singularities of Analytic Functions *567*
10.10 Singularities at Infinity *575*
10.11 Significance of Singularities *578*
10.12 Residues *580*
10.13 Evaluation of Real Definite Integrals *583*
10.14 Theorems on Limiting Contours *589*
10.15 Indented Contours *592*
10.16 Integrals Involving Branch Points *595*

11

Applications of Analytic Function Theory **622**

11.1 Introduction *622*
11.2 Inversion of Laplace Transforms *622*

11.3 Inversion of Laplace Transforms with Branch Points. The Loop Integral *625*
11.4 Conformal Mapping *628*
11.5 Applications to Two-Dimensional Fluid Flow *632*
11.6 Basic Flows *634*
11.7 Other Applications of Conformal Mapping *638*
11.8 The Schwarz–Christoffel Transformation *641*
11.9 Green's Functions and the Dirichlet Problem *652*
11.10 The Use of Conformal Mapping *658*
11.11 Other Two-Dimensional Green's Functions *661*

Answers to Problems **703**

Index **721**