PART 1. INITIAL VALUE PROBLEMS

CHAPTER	1. INTRODUCTION TO DISCRETE VARIABLE METHODS	
1.	Introduction	2
2.	Discrete variable methods	3
3.	Application to systems and higher order equations	12
4.	Sources of error	14
CHAPTER	2. CONVERGENCE AND STABILITY	
1.	Introduction	20
2.	Convergence	21
3.	Zero-stability	25
4.	Attainable order of zero stable linear multistep	
	methods	31
5.	Stability for fixed step-length	32
6.	Regions of absolute stability	34
7.	Strong stability	40
8.	Comparison of methods	41
9.	Relative stability	43
CHAPTER	3. ERROR ESTIMATION FOR INITIAL VALUE PROBLEMS	
1.	The analytic problem	45
2.	Discrete approximations	48
3.	The form of the local discretization error	52
4.	Estimation of local discretization error of one step	
	methods	56

5.	Comparison of error estimates for one step	
	methods	62
6.	Estimation of local discretization error for	
	predictor-corrector methods	63
7.	Local discretization error of hybrid methods	65
CHAPTER	4. GENERAL DISCUSSION OF IMPLEMENTATION PROBLEMS	
1.	Introduction	67
2.	Structure of a simple integrator	67
3.	Parameter list of a simple integrator	69
4.	More sophisticated programs	72
5.	Rounding errors	74
CHAPTER	5. RUNGE-KUTTA METHODS	
1.	Introduction	76
2.	Derivation of particular methods	76
3.	Methods with error estimates	79
4.	Local truncation error	81
5.	Optimal choice of step-size	82
6.	Optimal choice of order	83
7.	Assessments of methods	84
CHAPTER	6. IMPLEMENTATION OF LINEAR MULTISTEP METHODS	
1.	Introduction	86
2.	Why the Adams methods?	87
3.	Step-by-step algorithms	89
4.	Representation, step changing, error estimation	90
5.	Generalized algorithms	95
6.	Variable order, starting the integration	96
7.	Theory of variable step/variable order programs	100
CHAPTER	7. EXTRAPOLATION METHODS	
1.	Introduction	105
2.	Richardson extrapolation	106
3.	Rational extrapolation	108
4.	The Gragg-Bulirsch-Stoer method	110
5.	Implementation of recurrence relations	111
6.	Implementation of step control mechanism	113

CHAPTER	8. TESTING PROGRAMS AND COMPARISON OF METHODS				
1.	Introduction				
2.	Testing a program				
3.	Comparison of methods and programs	118			
4.	Conclusion	121			
	PART 2. STIFF PROBLEMS				
CHAPTER	9. INTRODUCTION TO STIFF PROBLEMS				
1.	Stiff initial value problems; stiffness ratio	123			
2.	Stability for stiff problems	125			
3.	Implicit methods	129			
4.	Step-size strategies	132			
5.	Illustrative example	133			
CHAPTER	10. IMPLICIT RUNGE-KUTTA AND RELATED METHODS				
1.	Limitations of explicit Runge-Kutta methods	136			
2.	Order conditions for Runge-Kutta methods	137			
3.	Optimal order implicit Runge-Kutta methods	141			
4.	Stability properties	145			
5.	Implementation of implicit Runge-Kutta methods	147			
6.	Semi-explicit and Rosenbrock methods	148			
CHAPTER	11. MULTISTEP METHODS FOR STIFF PROBLEMS				
1.	Implicit linear multistep methods	152			
2.	Backward differentiation formulae	154			
3.	Generalized multistep methods	158			
4.	Second derivative methods	161			
CHAPTER	12. EXTRAPOLATION METHODS FOR STIFF SYSTEMS AND A COM- OF METHODS FOR STIFF PROBLEMS	PARISON			
1.		164			
_•	Extrapolation methods for stiff ordinary differential				
-•	equations	164			
3.		169			
CHAPTER	13. NUMERICAL INTEGRATION OF SYSTEMS OF STIFF ORDINAR	Y			
ORAF LEK	DIFFERENTIAL EQUATIONS WITH SPECIAL STRUCTURE				
1.		174			
2.	Special structure	175			

3.	A partioned approximate inverse	1/8			
4.	Chemical reaction kinetic equations	180			
5.	Properties of reaction kinetic systems				
6.	Fast reactions associated with very reactive species				
7.	Applications	184			
8.	Local convergence of the parallel chord iteration	187			
9.	Equilibration and condition of F'	189			
10.	Numerical results	190			
CHAPTER	14. PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS				
1.	Introduction	197			
2.	Piecewise polynomials	198			
3.	Galerkin	199			
4.	Convergence of Galerkin approximations	201			
5.	Collocation	202			
6.	Method of lines	204			
7.	Conclusion	205			
	PART 3. BOUNDARY VALUE PROBLEMS				
CHAPTER	15. BOUNDARY VALUE PROBLEMS; FINITE DIFFERENCE METHOR	os			
1.	Introduction	208			
2.	Basic methods	209			
3.	Solution of the algebraic equations	211			
4.	Extrapolation	213			
5.	Deferred correction	214			
CHAPTER	16. SHOOTING METHODS FOR BOUNDARY VALUE PROBLEMS				
1.	Description of the basic method	216			
2.	Some examples of boundary-value problems	220			
3.	Solution of the matching equations	222			
4.	Parallel shooting methods	231			
5.	Invariant imbedding for obtaining estimates to				
	boundary values	236			
CHAPTER	17. LINEAR DIFFERENTIAL EQUATIONS AND DIFFERENTIAL				
	EIGENPROBLEMS				
1.	Introduction	239			
2.	Methods for linear boundary value problems	240			
3.	Linear differential eigenproblems	248			

CHAPTER	18. THE METHOD OF CONTINUATION AND GENERALIZED BOU	NDARY					
	VALUE PROBLEMS						
1.	Introduction						
2.	Problem of starting conditions						
3.	Conditions for convergence 255						
4.	Differential equations for the t-curve 257						
5.	Practical application	259					
6.	General parameter structure	261					
7.	Singularities and asymptotic solutions	264					
8.	Solution by the shooting method	266					
CHAPTER	19. EXPANSION METHODS						
1.	Introduction	269					
2.	Linear problems	269					
3.	Collocation methods	271					
4.	Residual minimizing techniques	272					
5.	Discretizations; uniform treatment of boundary						
	conditions	273					
6.	An example	275					
7.	Relations between methods 277						
8.	Error estimates; choice of functions h; 277						
9.	Extensions and conclusions 280						
į	PART 4. FUNCTIONAL DIFFERENTIAL PROBLEMS						
CHAPTER	20. INITIAL VALUE PROBLEMS FOR DELAY-DIFFERENTIAL	EQUATIONS					
1.	Introduction	283					
2.	Basic notions	284					
3.	Introduction to the use of numerical methods 286						
4.	Step-by-step formulae for equations with delay	289					
5.	Additional considerations and comments	294					
CHAPTER	21. INITIAL VALUE PROBLEMS FOR VOLTERRA INTEGRO-						
	DIFFERENTIAL EQUATIONS						
1.	Introduction	296					
2.	Numerical techniques 297						
3.	Methods of the first class 299						
4.	Methods of the second class 302						

	5. Block-by-block methods		305
	6.	Treatment of weak singularities	306
REFERENCES		ES	308
INDEX			331