

Contents

Translator's Note	v
Foreword	vii

Chapter I

The Kernel Theorem. Nuclear Spaces. Rigged Hilbert Space . . .	1
1. Bilinear Functionals on Countably Normed Spaces. The Kernel Theorem	2
1.1. Convex Functionals	3
1.2. Bilinear Functionals	7
1.3. The Structure of Bilinear Functionals on Specific Spaces (the Kernel Theorem)	11
Appendix. The Spaces K , S , and 2	20
2. Operators of Hilbert-Schmidt Type and Nuclear Operators . .	26
2.1. Completely Continuous Operators	27
2.2. Hilbert-Schmidt Operators	32
2.3. Nuclear Operators	37
2.4. The Trace Norm	47
2.5. The Trace Norm and the Decomposition of an Operator into a Sum of Operators of Rank 1	52
3. Nuclear Spaces. The Abstract Kernel Theorem	56
3.1. Countably Hilbert Spaces	57
3.2. Nuclear Spaces	62
3.3. A Criterion for the Nuclearity of a Space	66
3.4. Properties of Nuclear Spaces	71
3.5. Bilinear Functionals on Nuclear Spaces	73
3.6. Examples of Nuclear Spaces	79
3.7. The Metric Order of Sets in Nuclear Spaces	86
3.8. The Functional Dimension of Linear Topological Spaces	98

4. Rigged Hilbert Spaces. Spectral Analysis of Self-Adjoint and Unitary Operators	103
4.1. Generalized Eigenvectors	103
4.2. Rigged Hilbert Spaces	106
4.3. The Realization of a Hilbert Space as a Space of Functions, and Rigged Hilbert Spaces	110
4.4. Direct Integrals of Hilbert Spaces, and Rigged Hilbert Spaces	114
4.5. The Spectral Analysis of Operators in Rigged Hilbert Spaces	119
Appendix. The Spectral Analysis of Self-Adjoint and Unitary Operators in Hilbert Space.	127
1. The Abstract Theorem on Spectral Decomposition	127
2. Cyclic Operators	129
3. The Decomposition of a Hilbert Space into a Direct Integral Corresponding to a Given Self-Adjoint Operator	130

Chapter II

Positive and Positive-Definite Generalized Functions	135
1. Introduction	135
1.1. Positivity and Positive Definiteness	136
2. Positive Generalized Functions	142
2.1. Positive Generalized Functions on the Space of Infinitely Differentiable Functions Having Bounded Supports	142
2.2. The General Form of Positive Generalized Functions on the Space S	145
2.3. Positive Generalized Functions on Some Other Spaces	147
2.4. Multiplicatively Positive Generalized Functions	149
3. Positive-Definite Generalized Functions. Bochner's Theorem	151
3.1. Positive-Definite Generalized Functions on S	151
3.2. Continuous Positive-Definite Functions	152
3.3. Positive-Definite Generalized Functions on K	157
3.4. Positive-Definite Generalized Functions on Z	166
3.5. Translation-Invariant Positive-Definite Hermitean Bilinear Functionals	167
3.6. Examples of Positive and Positive-Definite Generalized Functions	169

4. Conditionally Positive-Definite Generalized Functions	175
4.1. Basic Definitions	175
4.2. Conditionally Positive Generalized Functions (Case of One Variable)	176
4.3. Conditionally Positive Generalized Functions (Case of Several Variables)	179
4.4. Conditionally Positive-Definite Generalized Functions on K	188
4.5. Bilinear Functionals Connected with Conditionally Positive-Definite Generalized Functions	189
Appendix	194
5. Evenly Positive-Definite Generalized Functions	196
5.1. Preliminary Remarks	196
5.2. Evenly Positive-Definite Generalized Functions on $S_{\frac{1}{2}}$	198
5.3. Evenly Positive-Definite Generalized Functions on $S_{\frac{1}{2}}$	211
5.4. Positive-Definite Generalized Functions and Groups of Linear Transformations	213
6. Evenly Positive-Definite Generalized Functions on the Space of Functions of One Variable with Bounded Supports	216
6.1. Positive and Multiplicatively Positive Generalized Functions	216
6.2. A Theorem on the Extension of Positive Linear Functionals	219
6.3. Even Positive Generalized Functions on Z	220
6.4. An Example of the Nonuniqueness of the Positive Measure Corresponding to a Positive Functional on Z_+	226
7. Multiplicatively Positive Linear Functionals on Topological Algebras with Involutions	229
7.1. Topological Algebras with Involutions	229
7.2. The Algebra of Polynomials in Two Variables	232

Chapter III

Generalized Random Processes	237
1. Basic Concepts Connected with Generalized Random Processes	237
1.1. Random Variables	237
1.2. Generalized Random Processes	242

1.3.	Examples of Generalized Random Processes	244
1.4.	Operations on Generalized Random Processes	245
2.	Moments of Generalized Random Processes. Gaussian Processes. Characteristic Functionals	246
2.1.	The Mean of a Generalized Random Process	246
2.2.	Gaussian Processes	248
2.3.	The Existence of Gaussian Processes with Given Means and Correlation Functionals	252
2.4.	Derivatives of Generalized Gaussian Processes	257
2.5.	Examples of Gaussian Generalized Random Processes	257
2.6.	The Characteristic Functional of a Generalized Random Process	260
3.	Stationary Generalized Random Processes. Generalized Random Processes with Stationary n th-Order Increments	262
3.1.	Stationary Processes	262
3.2.	The Correlation Functional of a Stationary Process	263
3.3.	Processes with Stationary Increments	265
3.4.	The Fourier Transform of a Stationary Generalized Random Process	268
4.	Generalized Random Processes with Independent Values at Every Point	273
4.1.	Processes with Independent Values	273
4.2.	A Condition for the Positive Definiteness of the Func- tional $\exp(\int f[\varphi(t)] dt)$	275
4.3.	Processes with Independent Values and Conditionally Positive-Definite Functions	279
4.4.	A Connection between Processes with Independent Values at Every Point and Infinitely Divisible Distribution Laws	283
4.5.	Processes Connected with Functionals of the n th Order	284
4.6.	Processes of Generalized Poisson Type	285
4.7.	Correlation Functionals and Moments of Processes with Independent Values at Every Point	286
4.8.	Gaussian Processes with Independent Values at Every Point	288

5. Generalized Random Fields	289
5.1. Basic Definitions	289
5.2. Homogeneous Random Fields and Fields with Homogeneous sth -Order Increments	290
5.3. Isotropic Homogeneous Generalized Random Fields	292
5.4. Generalized Random Fields with Homogeneous and Isotropic sth -Order Increments	294
5.5. Multidimensional Generalized Random Fields	297
5.6. Isotropic and Vectorial Multidimensional Random Fields	301

Chapter IV

Measures in Linear Topological Spaces	303
1. Basic Definitions	303
1.1. Cylinder sets	303
1.2. Simplest Properties of Cylinder Sets	305
1.3. Cylinder Set Measures	307
1.4. The Continuity Condition for Cylinder Set Measures	309
1.5. Induced Cylinder Set Measures	311
2. The Countable Additivity of Cylinder Set Measures in Spaces Adjoint to Nuclear Spaces	312
2.1. The Additivity of Cylinder Set measures	312
2.2. A Condition for the Countable Additivity of Cylinder Set Measures in Spaces Adjoint to Countably Hilbert Spaces	317
2.3. Cylinder Sets Measures in the Adjoint Spaces of Nuclear Countably Hilbert Spaces	320
2.4. The Countable Additivity of Cylinder Set Measures in Spaces Adjoint to Union Spaces of Nuclear Spaces	330
2.5. A Condition for the Countable Additivity of Measures on the Cylinder Sets in a Hilbert Space	333
3. Gaussian Measures in Linear Topological Spaces	335
3.1. Definition of Gaussian Measures	335
3.2. A Condition for the Countable Additivity of Gaussian Measures in the Conjugate Spaces of Countably Hilbert Spaces	339

4. Fourier Transforms of Measures in Linear Topological Spaces	345
4.1. Definition of the Fourier Transform of a Measure	345
4.2. Positive-Definite Functionals on Linear Topological Spaces	347
5. Quasi-Invariant Measures in Linear Topological Spaces	350
5.1. Invariant and Quasi-Invariant Measures in Finite- Dimensional Spaces	350
5.2. Quasi-Invariant Measures in Linear Topological Spaces.	354
5.3. Quasi-Invariant Measures in Complete Metric Spaces.	359
5.4. Nuclear Lie Groups and Their Unitary Representations. The Commutation Relations of the Quantum Theory of Fields	362
Notes and References to the Literature	371
Bibliography	377
Subject Index	381

