

Contents

Chapter I. Invariant Differential Equations. Lie's Theory

1. Introductory Remarks	1
2. Transformations	2
3. Transformation of Functions	3
4. Transformation of a Set of Curves	4
5. Transformation of a Direction Field	5
6. Differential Equations Invariant under Given Transformation	7
7. Lie's Integrating Factor	9
8. Infinitesimal Transformations.	12

Chapter II. Existence and Uniqueness

1. Preliminary Remarks	15
2. Metrization	16
3. Integral Operation	17
4. Convergence of Iterations	19
5. Application to Differential Equations	21
6. Dependence of Solutions on Initial Values or Other Parameters	21
7. Remarks about Other Existence Theorems . . .	25
8. Remarks about the Domain of Existence	26
9. Remarks about the Existence of Periodic Solutions	28
10. Remarks about Systems of Equations	30
11. Linear Differential Equations	33

Chapter III. Discussion of the Topology of Integral Curves

1. Singularities	37
2. Linear Differential Equations	38

3. Path Curves near a Singular Point: Geometrical Discussion	45
4. Existence of the Exceptional Integral Curve through a Singular Point	50
5. Integral Curves in the Large. Closed Integral Curves.	54
6. Number and Type of Singular Points	58

Chapter IV. Periodic Solutions of Differential Equations

1. Analytic Determination of Periodic Solutions	63
2. The Variational Equation	67
3. Holonomic Differential Equations	71
4. The Degenerate Problem	76
5. The Totally Degenerate Problem	82
6. The Bifurcation Problem for Equations of Second Order.	87
7. A Modified Implicit Function Theorem	96
8. Solution of the Degenerate Problem and of Related Problems	102
9. Remarks about Linear Differential Equations with Periodic Coefficients	103

Appendix to Chapters II, III, IV

1. Remarks about the Manifold of all Solutions of a Linear Differential Equation with Constant Coefficients.	110
2. Remarks about the Stability Problem	115

Chapter V. Linear Differential Equations for Analytic Functions

1. Introduction	119
2. Existence, Uniqueness, Circuits	121
3. Circuits	124
4. Regular Singularities	132
5. The Nature of the Proper and the Generalized Eigen-Solutions of a Differential Equation with a Regular Singularity	136
6. The Non-homogeneous Equation	142
7. An Equation of the n th Order for One Unknown Function	146

CONTENTS

ix

8. The Hypergeometric Functions	150
9. Integral Representation of the Hypergeometric Function	156
10. Singular Singularities. Pseudo-regular Differential Equations	160
11. Existence of a Pseudo-regular Solution	167
12. Asymptotic Expansion	176
13. Recessive and Dominant Solutions. Stokes Phenomenon	179
14. Asymptotic Expansion with Respect to a Parameter	188
15. Circuit Transformation of the Asymptotic Expansion with Respect to a Parameter	199