Contents

	Preface	xi
1	FINITE DIFFERENCES	
	1. Calculus to Algebra to Arithmetic	1
	2. Differentials and Finite Difference Approximations	3
	3. Finite Difference Schemes	5
	4. Accuracy Analysis	7
	5. Higher-Order Schemes	9
	Exercises	11
	Suggested Further Reading	11
2	TWO-POINT BOUNDARY VALUE PROBLEMS	
	1. Finite Difference Approximation of the Loaded String	10
	Equation	12
	2. Incorporation of Boundary Conditions	14
	3. Consistency and Stability: Convergence	16
	4. Higher-Order Consistency	18
	5. Finite Difference Approximation of the Beam Equation	19 21
	6. Splitting of the Beam Equation into Two String Equations	23
	7. Nonlinear Two-Point Boundary Value Problems	23
	Exercises	28
	Suggested Further Reading	20
3	VARIATIONAL FORMULATIONS	
	1. Energy Error	29
	2. Principle of Minimum Potential Energy	33
		vii

viii CONTENTS

	 More General Boundary Conditions Complementary Variational Principles Euler-Lagrange Equations Total Potential Energy of the Thin Elastic Beam Indefinite Variational Principles A Bound Theorem Exercises Suggested Further Reading 	36 38 40 42 43 45 45
4	FINITE ELEMENTS	
	 The Idea of Ritz Finite Element Basis Functions Finite Element Matrices Assembly of Global Matrices Essential and Natural Boundary Conditions Higher-Order Finite Elements Beam Element Complex Structures Exercises Suggested Further Reading 	48 51 53 55 59 61 67 67 69 70
5	DISCRETIZATION ACCURACY	
	 Energy Theorems Energy Rates of Convergence Sharpness of the Energy Error Estimate L₂ Error Estimate Error Estimate Richardson's Extrapolation to the Limit Numerical Integration Exercises Suggested Further Reading 	71 74 79 81 83 84 85 86 88
6	EIGENPROBLEMS	
	 Stability of Columns Vibration of Elastic Systems Finite Difference Approximation Rayleigh's Quotient Finite Element Approximation The Minmax Principle Discretization Accuracy of Eigenvalues Discretization Accuracy of Eigenfunctions Change of Basis: Condensation Numerical Integration: Lumping 	89 93 95 98 101 103 105 111 112

CONTENTS

	11.	Nonlinear Eigenproblems Exercises Suggested Further Reading	118 121 124
7		GEBRAIC PROPERTIES OF THE GLOBAL	
, .	1. 2. 3. 4. 5. 6. 7. 8. 9.	Eigenvalue Range in $Ky = \lambda My$ Spectral Norms of K and M Spectral Condition Numbers Irregular Meshes The Influence (Green's) Function Maximum Norms and Condition Numbers	125 128 131 135 138 141 143 150 152 154 155
8	1. 2. 3. 4.	Nonstationary Heat Transfer in a Rod Finite Difference Approximation Modal Analysis Finite Elements Essential Boundary Conditions Euler's Stepwise Integration in Time Explicit Finite Element Schemes Convergence of Euler's Method Stability Stable Time Step Size Estimate Numerical Example Implicit Unconditionally Stable Schemes Higher-Order Single Step Implicit Schemes Superstable Schemes Multistep Schemes Predictor-Corrector Methods Nonlinear Heat Condition and the Runge-Kutta Method Exercises Suggested Further Reading	158 160 161 163 163 165 167 168 169 171 173 175 177 179 182 187 189 190
9	EQ	UATION OF MOTION	
	1. 2. 3.	Spring-Mass System Single Step Explicit Scheme Conditionally Stable Schemes	193 194 196

~~	N N TF	$r_{F\lambda}$	1000
	3 A / 1		///

238

243

245

249

249

251

X		CONTENTS
4	Lattice of Springs and Masses	200
	5. Modal Decomposition	201
	5. Stability Conditions for $Ky + M\ddot{y} = 0$	203
	7. Nonlinear Equation of Motion	206
	3. Single Step Unconditionally Stable Implicit Scheme	208
	O. Unconditionally Stable Semiexplicit Schemes	211
10		214
	Runge-Kutta-Nyström Method	218
	2. Shooting in Boundary Value Problems	218
	Exercises	220
	Suggested Further Reading	228
10 u	VAVE PROPAGATION	
10 "		220
	1. Standing and Traveling Waves in a String	229
	2. Discretization in Space	232
	3. Spurious Dispersion	234
4	4. Effects of (Numerical) Viscosity	237

5. Higher-Order Elements

7. Flexural Waves in a Beam

Suggested Further Reading

6. Spurious Reflection

8. Stiff String

Exercises

Index 253