

Table of Contents

Preface

Foreword

| | | |
|------------------|--|----|
| Chapter 1 | Single and Double Hypergeometric Functions | |
| 1.1 | Introduction | 13 |
| 1.2 | The Integration of the Hypergeometric Differential Equation | 15 |
| 1.3 | Single Hypergeometric Functions of Higher Order | 19 |
| 1.4 | Appell's Double Hypergeometric Functions | 23 |
| 1.5 | The Kampé de Fériet Function | 29 |
| 1.6 | The Horn Functions | 36 |
| 1.7 | Horn's General Theory of Convergence of Double Hypergeometric Series | 37 |
| Chapter 2 | The Lauricella Functions | |
| 2.1 | Introduction and Definitions | 41 |
| 2.2 | Convergence of the Lauricella Series | 43 |
| 2.3 | Integral Representations of Euler Type | 48 |
| 2.4 | Integrals of Laplace Type | 49 |
| 2.5 | Integrals of Barnes Type | 51 |
| 2.6 | Integrals with Respect to Parameters | 54 |
| 2.7 | Other Integral Relations | 57 |
| 2.8 | Differential Properties - Addition and Multiplication Theorems | 61 |
| 2.9 | The General Theory of Convergence of Multiple Hypergeometric Series | 65 |
| 2.10 | The Triple Hypergeometric Functions of Lauricella-Saran | 66 |
| 2.11 | An Incomplete $F_D^{(n)}$ Function | 70 |
| 2.12 | Applications | 71 |
| Chapter 3 | Other Hypergeometric Functions of Several Variables | |
| 3.1 | The Srivastava Functions H_A , H_B and H_C | 74 |
| 3.2 | The Pandey Functions | 75 |
| 3.3 | The Quadruple Hypergeometric Functions | 77 |
| 3.4 | The Multiple Hypergeometric Functions $(k)_{E_D^{(n)}}$, $(k)_{E_D^{(n)}}$ and $(k)_{E_D^{(n)}}$ | 89 |
| 3.5 | The Generalised Horn Functions $(k)_{H_3^{(n)}}$ $(k)_{H_4^{(n)}}$ and $(k)_{H_2^{(n)}}$ | 97 |

| | | |
|------------------|--|-----|
| 3.6 | On the Functions $C_n^{(k)}$ and $D_{(n)}^{p,q}$ | 104 |
| 3.7 | Multiple Hypergeometric Functions of Higher Order | 106 |
| 3.8 | Applications | 112 |
| Chapter 4 | Multiple Hypergeometric Transformations | |
| 4.1 | Transformations of the Triple and Quadruple Hypergeometric Functions | 113 |
| 4.2 | Euler Integral Transformations of the Lauricella Functions | 121 |
| 4.3 | Transformations Obtainable from the Pochhammer Integrals | 124 |
| 4.4 | Transformations Obtainable from the Laplace Integrals | 126 |
| 4.5 | Special Transformations of ${}_{(1)}E_D^{(n)}$ and $D_{(n)}^{p,q}$ | 127 |
| 4.6 | The Elementary Manipulation of Series | 131 |
| 4.7 | Cases of Reducibility | 132 |
| 4.8 | A Generalisation of Bailey's Theorem | 139 |
| 4.9 | Certain Transformations Involving Multiple Hypergeometric Functions of Higher Order | 141 |
| Chapter 5 | Systems of Partial Differential Equations | |
| 5.1 | The Partial Differential Systems Associated with the Lauricella Functions | 148 |
| 5.2 | The Broad Nature of the General Integrals | 149 |
| 5.3 | The System Associated with F_1 . (I) | 152 |
| 5.4 | The System Associated with F_1 . (II) | 154 |
| 5.5 | The Application of Contour Integrals | 157 |
| 5.6 | The Integration of the System F_1 . (III) | 160 |
| 5.7 | The Integration of the System Associated with ϕ_2 | 163 |
| 5.8 | On the Systems $F_D^{(3)}$ and $F_D^{(4)}$ | 166 |
| 5.9 | The Intehroration of the System $F_D^{(n)}$ | 171 |
| 5.10 | On the System $\phi_2^{(n)}$ | 173 |
| Chapter 6 | Generating Functions and Recurrence Relations – Analytical Continuation | |
| 6.1 | Generating Relations of a General Nature | 179 |
| 6.2 | Applications to Lauricella Polynomials | 189 |
| 6.3 | Generating Relations of the Functions ${}_{(1)}E_D^{(k)}$, ${}_{(2)}E_D^{(k)}$ and ${}_{(1)}E_C^{(k)}$ and the Generalised Horn Functions ${}^{(k)}H_3^{(n)}$ and ${}^{(k)}H_4^{(n)}$ | 191 |

| | | |
|------------------------------|---|-----|
| 6.4 | Recurrence Relations for the Lauricella Functions | 193 |
| 6.5 | The Analytical Continuation of the Functions $F_C^{(n)}$, $F_A^{(n)}$ and $F_B^{(n)}$ | 197 |
| 6.6 | The Analytical Continuation of $F_D^{(n)}$ | 198 |
| 6.7 | The Analytical Continuation of $F_D^{(3)}$ | 200 |
| 6.8 | Certain Analytical Continuation Formulae for $F_D^{(n)}$ when n is unrestricted | 210 |
| 6.9 | Two Formulae of Toscano | 215 |
| 6.10 | The Function R of Carlson | 216 |
| Chapter 7 | Applications in Statistics | |
| 7.1 | Special Univariate Distributions | 219 |
| 7.2 | Special Multivariate Distributions | 222 |
| 7.3 | Applications to Bayesian Inference | 227 |
| 7.4 | Characteristic Functions | 230 |
| 7.5 | An Application in the Field of Genetics | 233 |
| 7.6 | The Distribution of a Sum of Gamma Variates | 237 |
| 7.7 | Distribution of the Ratio | 237 |
| Chapter 8 | Applications in Physics and Other Fields | |
| 8.1 | Applications to Ordinary Differential Equations | 246 |
| 8.2 | Asymptotic Solutions of Certain Differential Equations | 255 |
| 8.3 | Approximate Solutions of Differential Equations | 260 |
| 8.4 | A Type of Hyperelliptical Integral | 264 |
| 8.5 | Applications to Quantum Theory and Lie Theory | 266 |
| 8.6 | Applications to Dual Integral Equations | 273 |
| 8.7 | Integrals of Several Bessel Functions. Applications to the Limiting of Several Sinusoidal Signals with Gaussian Noise | 276 |
| 8.8 | Heat Conduction Applications | 278 |
| 8.9 | An Application in the Theory of Elasticity | 281 |
| Appendix A | Formulae Related to Multiple Hypergeometric Functions | 284 |
| Appendix B | Two Useful Computer Programs | 293 |
| Bibliography | | 296 |
| Index of Applications | | 305 |
| Index of Symbols | | 306 |
| Subject Index | | 309 |