

Contents

Chapter I. Introduction, Analytically Uniform Spaces, and Multiplicity Varieties

Summary	1
I.1 Introduction	2
I.2 Prerequisites	6
I.3 Analytically Uniform Spaces	8
I.4 Algebroid Functions and Multiplicity Varieties	14
Remarks	25
Problems	27

PART A. QUOTIENT STRUCTURE THEOREMS

Chapter II. The Geometric Structure of Local Ideals and Modules

Summary	29
II.1 L'Hospital Representation	29
II.2 Formulation of the Main Results	36
II.3 The Main Results for Principal Ideals	49
II.4 Conclusion of the Proofs of Theorems 2.3, 2.5, and 2.6	59
Remarks	70
Problems	71

Chapter III. Semilocal Theory

Summary	72
III.1 Formulation of the Main Results	72
III.2 L'Hospital Representation	77
III.3 Connectability	78

III.4	Principal Ideals	81
III.5	Completion of the Proofs	88
	Remarks	94
	Problems	94

Chapter IV. Passage from Local to Global

	Summary	95
IV.1	Localizeable Spaces	96
IV.2	Formulation of the Quotient Structure Theorem (Fundamental Principle)	98
IV.3	Cohomology with Growth Conditions	98
IV.4	Proof of the Main Result	101
IV.5	The Oka Embedding	116
	Remarks	119
	Problems	120

Chapter V. Examples

	Summary	121
V.1	Example 1. The Space \mathcal{H} of Entire Functions	122
V.2	Example 2. The space \mathcal{H}'	138
V.3	Example 3. The Space $\mathcal{D}_{F'}$ of Distributions of Finite Order	139
V.4	Example 4. The Space \mathcal{D}' of all Distributions	148
V.5	Example 5. The Space \mathcal{E} of Indefinitely Differenti- able Functions	152
V.6	Example 6. Non-Quasianalytic Classes	163
V.7	Example 7. Rapidly Increasing Functions and Distributions	169
V.8	Example 8. Formal Power Series and Polynomials	172
	Remarks	172
	Problems	173

PART B. SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS

Chapter VI. Inhomogeneous Equations

	Summary	175
VI.1	General Inhomogeneous Systems	175
VI.2	Fundamental Solutions	179
	Remarks	187
	Problems	187

Chapter VII. Integral Representation of Solutions of Homogeneous Equations

Summary	188
Remarks	196
Problems	197

Chapter VIII. Extension and Comparison Theorems. Elliptic and Hyperbolic Systems

Summary	198
VIII.1 Comparison Theorems	199
VIII.2 Elliptic Systems	204
VIII.3 Hyperbolic Systems	206
VIII.4 Quasihyperbolicity	210
Remarks	212
Problems	213

Chapter IX. General Theory of Cauchy's Problem

Summary	214
IX.1 Formulations of the Problems	217
IX.2 Generalization of the Cauchy-Kowalewski Theorem	224
IX.3 Hyperbolic Systems	231
IX.4 Parabolic Systems	231
IX.5 Groups and Semigroups Associated with Cauchy and Initial Value Problems	232
IX.6 Structure of Cauchy Data	235
IX.7 The Fundamental Solution and Nonlinear Initial Surfaces	240
IX.8 Domain of Dependence and Lacunas	243
IX.9 Uniqueness of the Cauchy Problem	244
IX.10 More Refined Uniqueness Results	265
IX.11 The Zeros of Solutions of Elliptic Systems	271
IX.12 General Systems	276
Remarks	276
Problems	279

Chapter X. Balayage and General Boundary Value Problems

Summary	282
X.1 Balayage, General Boundary Value Problems, and Interpolation	283
X.2 An Example	287
X.3 The General Case	293

X.4	The Dirichlet Problem	295
X.5	Convex Polyhedra	303
X.6	The Fredholm Alternative.	305
X.7	The Wiener-Hopf and Goursat Problems	309
X.8	The Newman-Shapiro Parametrization Problem	311
X.9	General Systems	314
	Remarks	314
	Problems	315

Chapter XI. Miscellanae

	Summary	316
XI.1	Extension to Convolution Systems	317
XI.2	Singularities	325
XI.3	General Domains	367
XI.4	Special Functions and Group Representations.	379
XI.5	Variable Coefficient Equations	385
	Remarks	388
	Problems.	391

PART C. SEQUENCES OF OPERATORS

Chapter XII. Lacunary Series. Refined Comparison Theorems

	Summary	394
XII.1	Formulation of the Problem	395
XII.2	$(\mathcal{W}, \mathcal{W}_1)$ Density	406
XII.3	Analytic $(\mathcal{W}, \mathcal{W}_1)$ Density	410
XII.4	Geometric Density	413
XII.5	Example 1. The Classical Fabry Gap Theorem	416
XII.6	Example 2. Analog of the Fabry Gap Theorem for \mathcal{E}	419
XII.7	Example 3. Relation to the Riemann ζ Function	422
XII.8	Example 4. An Analog of the Fabry Gap Theorem for Several Variables and Complex a^j	432
XII.9	Grouping of Terms	436
XII.10	Natural Boundaries	438
XII.11	The Converse Problem	439
	Remarks	442
	Problems.	445

Chapter XIII. General Theory of Quasianalytic Functions

	Summary	446
XIII.1	General Quasianalyticity and Approximation	447
XIII.2	A Nonsymmetric Generalization of the Denjoy- Carleman Classes	459

CONTENTS

xiii

XIII.3 The Punctual and Local Images of Quasi-analytic and Non-Quasianalytic Classes . . . 473

XIII.4 Quasianalytic Functions on Lines 484

 Remarks 486

 Problems 487

Bibliography 490

Index of Special Notation 497

Index 501