

Table of Contents

	Page
PART I: THE LAPLACE TRANSFORM	1
1. Definition and Elementary Properties	1
1.1. The Laplace Transform	1
1.2. Important Properties	3
1.3. Asymptotic Properties: Watson's lemma	8
2. The Inversion Theorem	15
2.1. The Riemann-Lebesgue Lemma	15
2.2. Dirichlet Integrals	17
2.3. The Inversion Integral	19
2.4. Inversion of Rational Functions	20
2.5. Taylor Series Expansion	23
3. Ordinary Differential Equations	26
3.1. First and Second Order Differential Equations	26
3.2. Higher Order Differential Equations	29
3.3. Simultaneous Differential Equations	33
3.4. Equations With Polynomial Coefficients	42
4. Partial Differential Equations	47
4.1. The Diffusion Equation	47
4.2. Wave Propagation	50
5. Integral Equations	59
5.1. Convolution Equations of Volterra Type	59
5.2. Convolution Equations Over an Infinite Range	64
5.3. The Percus-Yevick Equation for Hard Rods	67
6. The Inversion Integral	76
6.1. Inversion of Meromorphic Functions	76
6.2. Inversions Involving a Branch Point	79
6.3. Watson's Lemma for Loop Integrals	82

	Page
6.4. Asymptotic Forms for Large t	84
6.5. Heaviside Series Expansion	86
PART II: THE FOURIER TRANSFORM	
7. Definitions and Elementary Properties	89
7.1. The Exponential, Sine and Cosine Transforms	89
7.2. Important Properties	95
7.3. Spectral Analysis	97
7.4. Kramers-Krönig Relations	101
8. Application to Partial Differential Equations	110
8.1. Potential Problems	110
8.2. Water Waves: Basic Equations	114
8.3. Water Waves Generated by an Initial Surface Displacement	117
8.4. Waves Due to a Periodic Disturbance: Radiation Condition	120
9. Generalized Functions	130
9.1. The Delta Function	130
9.2. Test Functions and Generalized Functions	131
9.3. Elementary Properties	136
9.4. Analytic Functionals	143
9.5. Fourier Transforms of Generalized Functions	145
10. Green's Functions	155
10.1. One Dimensional Green's Functions	155
10.2. Green's Functions as Generalized Functions	159
10.3. Poisson's Equation in Two Dimensions	162
10.4. Helmholtz's Equation in Two Dimensions	168
11. Fourier Transforms in Two or More Variables	178
11.1. Basic Notation and Results	178
11.2. Diffraction of Scalar Waves	182
11.3. Retarded Potentials of Electromagnetism	185

PART III: OTHER IMPORTANT TRANSFORMS

	Page
12. Mellin Transforms	195
12.1. Definitions	195
12.2. Simple Examples	196
12.3. Elementary Properties	201
12.4. Potential Problems in Wedge Shaped Regions	203
12.5. Transforms Involving Polar Coordinates	204
12.6. Hermite Functions	207
13. Mellin Transforms in Summation	214
13.1. Mellin Summation Formula	214
13.2. A Problem of Ramanujan	216
13.3. Asymptotic Behavior of Power Series	219
14. Integrals Involving a Parameter	225
14.1. Preliminary Example	225
14.2. Another Example	226
14.3. Ascending Expansions for Fourier Integrals	227
14.4. Multidimensional Integrals	229
15. Hankel Transforms	237
15.1. The Hankel Transform Pair	237
15.2. Elementary Properties	240
15.3. Some Examples	242
15.4. Boundary Value Problems	243
15.5. Weber's Integral	245
16. Dual Integral Equations	254
16.1. The Electrified Disc	254
16.2. Dual Integral Equations of Titchmarsh Type	255
16.3. Erdelyi-Köber Operators	258

	Page
17. Integral Transforms Generated by Green's Functions	267
17.1. The Basic Formula	267
17.2. Finite Intervals	269
17.3. Some Singular Problems	272
17.4. Kontorovich-Lebedev Transform	276
17.5. Boundary Value Problem in a Wedge	278
17.6. Diffraction of a Pulse by a Two-Dimensional Half-Plane	280
 PART IV: SPECIAL TECHNIQUES	
18. The Wiener-Hopf Technique	288
18.1. The Sommerfeld Diffraction Problem	289
18.2. The Wiener-Hopf Procedure: Half-plane Problems	299
18.3. Integral and Integro-Differential Equations	301
19. Methods Based on Cauchy Integrals	313
19.1. Wiener-Hopf Decomposition by Contour Integration	313
19.2. Cauchy Integrals	315
19.3. The Discontinuity Theorem	320
19.4. The Riemann Hilbert Problem	321
19.5. Simple Applications	323
19.6. Problems in Linear Transport Theory	324
19.7. The Albedo Problem	329
19.8. A Diffraction Problem	332
20. Laplace's Method for Ordinary Differential Equations	342
20.1. Integral Transform Solutions	342
20.2. Hermite Polynomials	344
20.3. Hermite Functions	347

20.4.	Bessel Functions: Integral Representations	351
20.5.	Bessel Functions of the First Kind	353
20.6.	Functions of the Second and Third Kinds	356
20.7.	Poisson and Related Representations	362
20.8.	Modified Bessel Functions	364
21.	Numerical Inversion of Laplace Transforms	374
21.1.	Gaussian Quadrature Formulas for the Laplace Inversion Integral	374
21.2.	Approximation of $F(p)$ by Chebyshev Polynomials for Real p	379
21.3.	Approximation of $f(t)$ by Orthogonal Polynomials	382
21.4.	Padé Approximation	385
21.5.	Rational Approximation of $F(p)$	388
APPENDICES		
A:	The Factorial Function	396
B:	Riemann's Zeta Function	399
C:	The Exponential Integral	402
BIBLIOGRAPHY		403
INDEX		406

