CONTENTS

1 VARIATIONAL PROBLEMS IN AN ABSTRACT SETTING 1

1.1. Introduction 1
1.2. Typical Problems 3
1.3. Basic Functional Analysis 3
1.4. General Functional Analysis for Minimization Problems 7
1.5. The Role of Convexity 13
1.6. Minimizing Sequences 17

2 THEORY OF DISCRETIZATION 28

2.1. Introduction 28
2.2. Constrained Minimization 28
2.3. Unconstrained Minimization 31
2.4. Remarks on Operator Equations 34

3 EXAMPLES OF DISCRETIZATION 37

3.1. Introduction 37
3.2. Regularization 37
3.3. A Numerical Method for Optimal Control Problems 41
3.4. Chebyshev Solution of Differential Equations 52
3.5. Calculus of Variations 55
3.6. Two-Point Boundary Value Problems 59
3.7. The Ritz Method 65

4 GENERAL BANACH-SPACE METHODS OF GRADIENT TYPE **70**

4.1. Introduction 70
4.2. Criticizing Sequences and Convergence in General 71
4.3. Global Minimum along the Line 76
4.4. First Local Minimum along the Line: Positive Weight Thereon 77
4.5. A Simple Interval along the Line 80
4.6. A Range Function along the Line 84
4.7. Search Methods along the Line 89
4.8. Specialization to Steepest Descent 93
4.9. Step-Size Algorithms for Constrained Problems 95
4.10. Direction Algorithms for Constrained Problems 105
4.11. Other Methods for Constrained Problems 110

5 CONJUGATE-GRADIENT AND SIMILAR METHODS IN HILBERT SPACE **114**

5.1. Introduction 114
5.2. Conjugate Directions for Quadratic Functionals 114
5.3. Conjugate Gradients for Quadratic Functionals 117
5.4. Conjugate Gradients as an Optimal Process 119
5.5. The Projected-Gradient Viewpoint 122
5.6. Conjugate Gradients for General Functionals 125
5.7. Local-Convergence Rates 127
5.8. Computational Modifications 136

6 GRADIENT METHODS IN $\mathbb{R}^l$ **142**

6.1. Introduction 142
6.2. Convergence of $x_{n+1} - x_n$ to Zero 142
6.3. The Limit Set of $\{x_n\}$ 145
6.4. Improved Convergence Results 150
6.5. Constrained Problems 154
6.6. Minimization along the Line 154

7 VARIABLE-METRIC GRADIENT METHODS IN $\mathbb{R}^l$ **159**

7.1. Introduction 159
7.2. Variable-Metric Directions 159
7.3. Exact Methods for Quadratics 165
7.4. Some Particular Methods 168
7.5. Constrained Problems 178

8 OPERATOR-EQUATION METHODS **180**

8.1. Introduction 180
8.2. Newton and Newton-like Methods 181
8.3. Generalized Linear Iterations 185
8.4. Least-Squares Problems 190

9 AVOIDING CALCULATION OF DERIVATIVES **194**

9.1. Introduction 194
9.2. Modifying Davidon's First Method 194
9.3. Modifying Newton's Method 197
9.4. Modifying the Gauss-Newton Method 200
9.5. Modifying the Gauss-Newton-Gradient Compromise 201
9.6. Methods Ignoring Derivatives 203

EPILOGUE **211**

REFERENCES **213**

INDEX **225**