

Contents

<i>Preface</i>	ix
1. An Introduction to General Linear Integral Equations	1
1.1 Examples of Integral Equations in Simple Practical Problems	2
1.2 Classification of Equations	5
1.3 Abel's Integral Equation	6
1.4 Extensions of Abel's Approach	8
Problems	9
2. Fredholm Integral Equations and the Fredholm Alternative	11
2.1 Square-integrable Functions and Kernels	11
2.2 General Remarks about Kernels of Finite Rank	13
2.3 Equations of the First Kind with Degenerate Kernels	15
2.4 Equations of the Second Kind with Degenerate Kernels	17
2.5 The Fredholm Alternative	20
Problems	21
3. Schmidt Theory and the Resolvent Kernel	23
3.1 The Neumann Series	23
3.2 The Fredholm Identities	27
3.3 The Nature of Schmidt's Method	29
3.4 Application of Schmidt's Approach	30
3.5 Theoretical Investigation of the Resolvent Kernel	33
3.6 The Fredholm Alternative for Arbitrary \mathcal{L}^2 Kernels	36
Problems	40
4. Fredholm Formulas and the Resolvent Equation	42
4.1 Fredholm-Carleman Theory	42
4.2 The Factorization of the Modified Fredholm Determinant	46
4.3 Various Deductions from the Factorization Formula	49
4.4 The Resolvent Equation	51
4.5 A Simple Application	54
Problems	54

5.	The Nature of Volterra Equations	57
5.1	Volterra Kernels	57
5.2	Volterra Equations with Difference Kernels	58
5.3	Degenerate Volterra Kernels of Rank 1	60
5.4	Elementary Perturbation Theory for Volterra Kernels	61
5.5	Positive Kernels and Comparison Theorems	62
5.6	Nonlinear Volterra Integral Equations	64
5.7	The Uniqueness of Solutions of Boundary-value Problems	67
5.8	Volterra Equations of the First Kind	70
	Problems	70
6.	Characteristic Behavior of Integral Equations	73
6.1	Explicit Construction of Characteristic Functions	73
6.2	Decomposition of a Kernel in Terms of Characteristic Functions	75
6.3	Singular Decomposition of the Resolvent Kernel	76
6.4	The Rank of Characteristic Values	79
	Problems	83
7.	Hermitian Kernels and Their Characteristic Values	85
7.1	The Nature of Hermitian Kernels	85
7.2	Elementary Proofs of the Existence of Characteristic Values	89
7.3	Estimation of Characteristic Values using Trace Relations	91
7.4	Schmidt's Characteristic-value Existence Proof	93
7.5	Schmidt Estimates for the Least Characteristic Value	97
	Problems	99
8.	Other Existence Proofs for the Characteristic Values of Hermitian Kernels	102
8.1	Introductory Remarks	102
8.2	Characteristic Values and Functions by Iteration	103
8.3	An Existence Proof Based upon an Extremal Property	106
8.4	Extremal Characterization of Larger Characteristic Values	110
8.5	The Extension to Negative Characteristic Values	112
	Problems	113
9.	The Rayleigh-Ritz Procedure and Related Results	116
9.1	Estimation Using the Rayleigh Quotient	116
9.2	Courant's Approach to the Estimation Problem	118
9.3	Independent Investigation of the Larger Characteristic Values	119
9.4	A Generalized Rayleigh-Ritz Procedure	122
9.5	Some Useful Inequalities	126
	Problems	128
10.	Error Bounds for the Characteristic Values and Characteristic Functions of Hermitian Kernels	131
10.1	The General Quality of Approximation	131
10.2	Another Look at the Courant Procedure	132
10.3	The Nature of Rayleigh-Ritz Approximations to Characteristic Functions	134
10.4	Kellogg's Approach Once More	137

10.5	Error Bounds for Iterative Approximations to Characteristic Functions	141
10.6	A Classical Example	144
	Problems	145
11.	Intermediate Operators and the Method of Weinstein and Aronszajn	147
11.1	Lower Bounds (Once over Lightly)	147
11.2	Intermediate Comparison Operators	148
11.3	Theoretical Basis for the Weinstein-Aronszajn Method	149
11.4	Classical Construction Procedures	152
11.5	The Method of Special Choices	154
11.6	Lower Bounds by Truncation of the Base Operator	156
11.7	General Remarks	158
	Problems	159
12.	Parametric Dependence of Characteristic Values and Characteristic Functions	162
12.1	On the Nature of Regular Perturbations	162
12.2	The Analyticity of the Resolvent	163
12.3	The Algebraic Dependence of the Characteristic Values	165
12.4	The Special Case of Hermitian K	168
12.5	The Analyticity of Characteristic Functions of Semisimple Kernels	171
12.6	Two-parameter Perturbations	174
12.7	Applications	177
	Problems	180
13.	Series Expansions and Their Convergence	185
13.1	Generalized Fourier Series and Bessel's Inequality	185
13.2	Hilbert's Theorem and a Completeness Criterion	187
13.3	Series Developments for Hermitian Kernels and Their Resolvents	191
13.4	Continuous Kernels and Mercer's Theorem	195
13.5	The Weinstein-Kato Method of Characteristic-value Approximation	200
	Problems	205
14.	Expansion Theorems for General Kernels	210
14.1	Hilbert-Schmidt Kernels	210
14.2	Singular Functions and Singular Values	212
14.3	Series Expansions in Terms of Singular Functions	214
14.4	Normal Kernels	217
14.5	The Inequalities of Schur and Carleman	222
	Problems	228
15.	Nuclear Kernels, Composite Kernels, and the Classes C_p	231
15.1	Nuclear Kernels and the Trace Class	231
15.2	Composite Kernels	236
15.3	Trace Identities for Composite Kernels	239
15.4	The Results of Chang and Weyl	243
15.5	Kernels of the Class C_p	245
	Problems	248

16.	The Effect of Smoothness Conditions on General Kernels	251
16.1	Hölder Continuity	251
16.2	Bounded Variation	255
16.3	Kernels with Combined Properties	257
16.4	Differentiable Kernels	261
16.5	Analogies with Fourier Series	265
	Problems	267
17.	Other Types of Special Kernels	269
17.1	General Decomposition of an \mathcal{L}^2 Kernel	269
17.2	Anti-Hermitian Kernels	270
17.3	Symmetrizable Kernels	272
17.4	Complex-Symmetric Kernels and Variational Principles	278
17.5	Positive Kernels	283
	Problems	289
18.	Difference Kernels and the Method of Wiener and Hopf	292
18.1	The Infinite Interval	292
18.2	The Approach of Wiener and Hopf	295
18.3	Generalizations of the Wiener-Hopf Procedure	299
18.4	Difference Kernels on a Finite Interval	301
	Problems	305
Appendix A	The Correspondence between Differential and Integral Equations	307
Appendix B	Relevant Fundamental Concepts	313
References		325
<i>Name Index</i>		351
<i>Subject Index</i>		357
<i>Index of Symbols</i>		369

