CONTENTS

PREFACE xi

Introduction 1

I.1 Definitions and Examples 1

Chapter 1 THE DIFFUSION EQUATION 7

1.1 Derivation 7
1.2 Problems 9
1.3 Simple Solutions 10
1.4 Problems 12
1.5 Series Solutions 13
1.6 Problems 16
1.7 Nonhomogeneous End Conditions 17
1.8 Problems 18
1.9 The Maximum Principle 19
1.10 Problems 20

Chapter 2 LAPLACE TRANSFORM METHODS 23

2.1 Introductory Example 24
2.2 Problems 25
2.3 A Finite Interval Problem 27
2.4 Problems 29
2.5 Delta Function 30
2.6 Problems 33
2.7 Supplementary Problems 34

Chapter 3 THE WAVE EQUATION 35

3.1 Derivation 35
3.2 Problems 36
3.3 An Infinite-Interval Problem 38
3.4 Problems 42
3.5 Series Solutions 45
3.6 Problems 46
3.7 A Problem with Radial Symmetry 48
3.8 Problems 49
3.9 Transforms 50
3.10 Problems 51
3.11 Uniqueness 52
3.12 Supplementary Problems 53

Chapter 4 THE POTENTIAL EQUATION 55

4.1 Laplace's and Poisson's Equations 55
4.2 Problems 58
4.3 Simple Properties of Harmonic Functions 59
4.4 Some Special Solutions—Series 62
4.5 Problems 64
4.6 Discontinuous Boundary Data 66
4.7 Complex Variables and Conformal Mapping 68
4.8 Problems 72

Chapter 5 CLASSIFICATION OF SECOND-ORDER EQUATIONS 75

5.1 Cauchy Data on y-Axis 75
5.2 Cauchy Data on Arbitrary Curve 77
5.3 Problems 78
5.4 Case I: $B^2 - AC > 0$ 79
5.5 Case II: $B^2 - AC = 0$ 81
5.6 Case III: $B^2 - AC < 0$ 82
5.7 Problems 83
5.8 Discontinuities; Signal Propagation 86
5.9 Problems 87
5.10 Some Remarks 89

Chapter 6 FIRST-ORDER EQUATIONS 91

6.1 Linear Equation Examples 91
6.2 Problems 94
6.3 Quasi-Linear Case 95
6.4 Problems 97

6.5 Further Properties of Characteristics 99
6.6 Problems 101
6.7 More Variables 101

Chapter 7 EXTENSIONS 103

7.1 More Variables 103
7.2 Problems 107
7.3 Series and Transforms 112
7.4 Problems 115
7.5 Legendre Functions 116
7.6 Problems 121
7.7 Spherical Harmonics 121
7.8 Problems 125

Chapter 8 PERTURBATIONS 127

8.1 A Nonlinear Problem 127
8.2 Problems 130
8.3 Two Examples from Fluid Mechanics 130
8.4 Boundary Perturbations 134
8.5 Problems 135

Chapter 9 GREEN'S FUNCTIONS 139

9.1 Some Consequences of the Divergence Theorem 139
9.2 The Laplacian Operator 142
9.3 Problems 144
9.4 Potentials of Volume and Surface Distributions 147
9.5 Problems 150
9.6 Modified Laplacian 152
9.7 Problems 154
9.8 Wave Equation 156
9.9 Problems 158

Chapter 10 VARIATIONAL METHODS 161

10.1 A Minimization Problem 161
10.2 Problems 164
10.3 Natural Boundary Conditions 167
10.4 Subsidiary Conditions 168
10.5 Problems 172
10.6 Approximate Methods 174
10.7 Problems 178
10.8 Finite-Element Method 180
10.9 Supplementary Problems 183

Chapter 11 EIGENVALUE PROBLEMS 189

11.1 A Prototype Problem 189
11.2 Some Eigenvalue Properties 190
11 3 Problems 193
11.4 Perturbations 197
11.5 Approximations 199
11.6 Problems 201

Chapter 12 MORE ON FIRST-ORDER EQUATIONS 203

12.1 Envelopes 203
12.2 Characteristic Strips 205
12.3 Complete Integral 209
12.4 Problems 212
12.5 Legendre Transformation 214
12.6 Problems 215
12.7 Propagation of a Disturbance 217
12.8 Complete Integral and Eikonal Function 220
12.9 Hamilton–Jacobi Equation 222
12.10 Problems 224

Chapter 13 MORE ON CHARACTERISTICS 227

13.1 Discontinuities—A Preliminary Example 227
13.2 Weak Solutions 230
13.3 Burgers' Equation 233
13.4 Problems 236
13.5 A Compressible Flow Problem 239
13.6 A Numerical Approach 242
13.7 Problems 246
13.8 More Dependent Variables 250
13.9 More Independent Variables 253
13.10 Problems 255

Chapter 14 FINITE-DIFFERENCE EQUATIONS AND NUMERICAL METHODS 257

14.1 Accuracy and Stability; A Diffusion Equation Example 257
14.2 Error Analysis 259
14.3 Problems 262
14.4 More Dimensions, or Other Complications 264
14.5 Series Expansions 267
14.6 Problems 269
14.7 Wave Equation 270
14.8 A Nonlinear Equation 272

14.9 Problems 273
14.10 Boundary Value Problems 274
14.11 Problems 278
14.12 Series; Fast Fourier Transform 278
14.13 Problems 281

Chapter 15 SINGULAR PERTURBATION METHODS 285

15.1 A Boundary Layer Problem 285
15.2 A More General Procedure 287
15.3 Problems 289
15.4 A Transition Situation 292
15.5 Problems 295
15.6 Asymptotic Analysis of Wave Motion 297
15.7 Boundary Layer near a Caustic 300
15.8 Problems 303
15.9 Multiple Scaling 303
15.10 Problems 306

References 309

INDEX 313