

CONTENTS

<i>Preface</i>	v
CH. 1. INTRODUCTION	1
1.1. What is asymptotics?	1
1.2. The O-symbol	3
1.3. The o-symbol.	10
1.4. Asymptotic equivalence	10
1.5. Asymptotic series	11
1.6. Elementary operations on asymptotic series	14
1.7. Asymptotics and Numerical Analysis	18
1.8. Exercises	19
CH. 2. IMPLICIT FUNCTIONS	21
2.1. Introduction	21
2.2. The Lagrange inversion formula	22
2.3. Applications	23
2.4. A more difficult case	25
2.5. Iteration methods	28
2.6. Roots of equations	30
2.7. Asymptotic iteration	31
2.8. Exercises	33
CH. 3. SUMMATION	34
3.1. Introduction	34
3.2. Case <i>a</i>	34
3.3. Case <i>b</i>	36
3.4. Case <i>c</i>	37
3.5. Case <i>d</i>	38

3.6.	The Euler-Maclaurin sum formula	40
3.7.	Example	42
3.8.	A remark	42
3.9.	Another example	43
3.10.	The Stirling formula for the Γ -function in the complex plane	46
3.11.	Alternating sums	49
3.12.	Application of the Poisson sum formula	52
3.13.	Summation by parts	56
3.14.	Exercises	58
CH. 4.	THE LAPLACE METHOD FOR INTEGRALS	60
4.1.	Introduction	60
4.2.	A general case.	63
4.3.	Maximum at the boundary	65
4.4.	Asymptotic expansions	66
4.5.	Asymptotic behaviour of the Γ -function	69
4.6.	Multiple integrals	71
4.7.	An application	72
4.8.	Exercises	75
CH. 5.	THE SADDLE POINT METHOD	77
5.1.	The method.	77
5.2.	Geometrical interpretation	79
5.3.	Peakless landscapes.	82
5.4.	Steepest descent	83
5.5.	Steepest descent at end-point	86
5.6.	The second stage	86
5.7.	A general simple case	87
5.8.	Path of constant altitude	89
5.9.	Closed path	90
5.10.	Range of a saddle point	91
5.11.	Examples.	93
5.12.	Small perturbations	96
5.13.	Exercises	101

CH. 6. APPLICATIONS OF THE SADDLE POINT METHOD	102
6.1. The number of class-partitions of a finite set . . .	102
6.2. Asymptotic behaviour of d_n	104
6.3. Alternative method	108
6.4. The sum $S(s, n)$	109
6.5. Asymptotic behaviour of P	112
6.6. Asymptotic behaviour of Q	115
6.7. Conclusions about $S(s, n)$	118
6.8. A modified Gamma Function	119
6.9. The entire function $G_0(s)$	123
6.10. Conclusions about $G(s)$	131
6.11. Exercises	133
CH. 7. INDIRECT ASYMPTOTICS	134
7.1. Direct and indirect asymptotics	134
7.2. Tauberian theorems	137
7.3. Differentiation of an asymptotic formula	139
7.4. A similar problem	141
7.5. Karamata's method	143
7.6. Exercises	147
CH. 8. ITERATED FUNCTIONS	148
8.1. Introduction	148
8.2. Iterates of a function	148
8.3. Rapid convergence	151
8.4. Slow convergence	153
8.5. Preparation	154
8.6. Iteration of the sine function	157
8.7. An alternative method	160
8.8. Final discussion about the iterated sine	164
8.9. An inequality concerning infinite series	166
8.10. The iteration problem	169
8.11. Exercises	175

CH. 9. DIFFERENTIAL EQUATIONS 176

9.1. Introduction 176

9.2. A Riccati equation. 177

9.3. An unstable case 184

9.4. Application to a linear second-order equation . . 186

9.5. Oscillatory cases 189

9.6. More general oscillatory cases 195

9.7. Exercises 198

INDEX 199

