CONTENTS

Introduction 1

CHAPTER ONE
The Riccati Equation

1. Introduction 7
2. Newton-Raphson Method 7
3. Multidimensional Version 11
4. Square Roots 12
5. Quasilinearization 13
6. The Riccati Equation 14
7. The First Order Linear Equation 15
8. Solution of Riccati Equation in Terms of Maximum Operation 15
9. Upper and Lower Bounds 18
10. Successive Approximations via Quasilinearization . . . 19
11. Monotonicity 20
12. Discussion 21
13. A Fundamental Lemma 22

Comments and Bibliography 22

CHAPTER TWO
Two-Point Boundary-Value Problems for Second-Order Differential Equations

1. Introduction 27
2. Two-Point Boundary-Value Problems for Second-Order Linear Differential Equation 28
3. The Inhomogeneous Equation 29

4. Vector-Matrix Approach 32
5. Green's Functions 34
6. Convexity 34
7. Quasilinearization 36
8. Discussion 37
9. Existence and Boundedness 37
10. Convergence 38
11. Convergence of the Picard Algorithm 40
12. A Numerical Example 41
13. General Second-Order Nonlinear Differential Equations . 42
14. A Numerical Example 43
15. Calculus of Variations 44
16. Quasilinearization 46
17. Upper and Lower Bounds via Quasilinearization . . . 48
18. Solution of Inner Problem 50
19. Further Uses of Quasilinearization 50
20. Dynamic Programming 51
21. Invariant Imbedding 52
22. Combination of Dynamic Programming and Quasilinearization 53
Comments and Bibliography 53

CHAPTER THREE

Monotone Behavior and Differential Inequalities

1. Monotonicity 57
2. An Elementary Approach 58
3. Relations Between Solutions and Coefficients 59
4. A Useful Factorization of the Second-Order Linear Differential Operator 60
5. A Positivity Result 62
6. A Related Parabolic Partial Differential Equation . . . 63
7. Characteristic Values Again 66
8. Variational Approach 67
9. Discussion 68
10. Improvement of Convergence Estimates 70
11. Discussion 71
Comments and Bibliography 71

CHAPTER FOUR

Systems of Differential Equations, Storage and Differential Approximation

1. Introduction 75
2. Quasilinearization Applied to Systems 76
3. Solution of Linear System 78
4. A Numerical Example 79
5. Multipoint Boundary-Value Problems 80
6. Successive Approximations and Storage 80
7. Simultaneous Calculation of Approximations 81
8. Discussion 83
9. Back and Forth Integration 85
10. Differential-Difference Equations 85
11. Reduction to Differential Equations. 86
12. Functional Differential Equations 87
13. Differential Approximation 88
14. Simple Version of Differential Approximation 89
15. A Renewal Equation 91
16. Discussion 93
17. Storage and Memory 95
18. Monotonicity for Systems 95
19. Monotonicity for *N*th-Order Linear Differential Equations . 97
20. Discussion 98
21. Use of the Gram-Schmidt Orthogonalization Procedure in the Numerical Solution of Linear Boundary-Value Problems 98
22. Dynamic Programming 103
23. Invariant Imbedding 104
Comments and Bibliography 105

CHAPTER FIVE

Partial Differential Equations

1. Introduction 111
2. Parabolic Equations 112
3. A Specific Equation 112
4. Difference Approximation 113
5. Comparison Between Picard and Quasilinearization . . . 114
6. Gradient Techniques 114

7. An Elliptic Equation 116
8. Computational Aspects 117
9. Positivity and Monotonicity 121
10. The Hopf-Lax Equation 122
Comments and Bibliography 123

CHAPTER SIX
Applications in Physics, Engineering, and Biology

1. Introduction 125
2. Optimal Design and Control 125
3. An Example 127
4. Numerical Results 129
5. An Inverse Problem in Radiative Transfer 130
6. Analytic Formulation 132
7. Numerical Results 132
8. The Van der Pol Equation 134
9. Orbit Determination as a Multipoint Boundary-Value Problem 137
10. Numerical Results 138
11. Periodogram Analysis 139
12. Numerical Results 140
13. Discussion 144
14. Periodic Forcing Terms 144
15. Estimation of Heart Parameters 144
16. Basic Assumptions 146
17. The Inverse Problem 147
18. Numerical Experiments 148
Comments and Bibliography 150

CHAPTER SEVEN
Dynamic Programming and Quasilinearization

1. Introduction 153
2. A Basic Functional Equation 153
3. Approximation in Policy Space 154
4. Dynamic Programming and Quasilinearization 156
5. Functional Equations 157
6. Discussion 158

7. Dynamic Programming and Differential Approximation . . 158
8. Dynamic Programming and the Identification of Systems . 159
9. Discussion 161
Comments and Bibliography 161

APPENDICES

One: Minimum Time Program 163
Two: Design and Control Program 167
Three: Program for Radiative Transfer—Inverse Problem for Two Slabs, Three Constants 183
Four: Van Der Pol Quation Program 185
Five: Orbit Determination Program 189
Six: Cardiology Program 193

Index 203