

CONTENTS

Chapter 1 Fundamental Concepts

1

- 1 Introduction, 1
- 2 A Simple Physical Model, 2
- 3 A Classical Approach, 2
- 4 The Invariant Imbedding Approach, 6
- 5 Some Comments, Criticisms, and Questions, 9
- 6 A Minor Variant of the Model of Section 2, 9
- 7 A Major Variant of the Model of Section 2, 10
- 8 The Classical Approach Extended, 11
- 9 The Invariant Imbedding Approach Extended, 13
- 10 Some Comments on Possible Uses of the Reflection and
Transmission Functions, 15
- 11 Summary, 16
Problems, 17
References, 20

Chapter 2 Additional Illustrations of the Invariant Imbedding Method

22

- 1 Introduction, 22
- 2 A Non-Linear Problem, 23
- 3 A Generalization of the Model, 24
- 4 Invariant Imbedding Formulation of the Model in
Section 3, 25
- 5 The Linear Problem Revisited, 27
- 6 A Perturbation Approach, 28
- 7 Some Remarks and Comments, 31
- 8 The Riccati Transformation Method, 32
- 9 Summary, 35
Problems, 35
References, 38

Chapter 3 Functional Equations and Related Matters	39
1 Introduction,	39
2 A Basic Problem,	39
3 The Basic Functional Equations,	40
4 Some Applications of the Results of Section 3,	41
5 Differential Equations Via Functional Equations,	45
6 Summary,	47
Problems,	47
References,	53
Chapter 4 Existence, Uniqueness, and Conservation Relations	54
1 Introduction,	54
2 The “Physics” of the Conservative Case and Its Generalizations,	55
3 Another Derivation of the Reflection Function,	56
4 Some Conservation Relations,	57
5 Proof of Existence in the Conservative Case,	59
6 The Nonconservative Case: The Dissipation Function,	60
7 The Existence Proof,	63
8 Summary,	64
Problems,	65
References,	66
Chapter 5 Random Walk	67
1 Introduction,	67
2 A One-Dimensional Random Walk Process,	67
3 A Classical Formulation,	68
4 An Invariant Imbedding Formulation,	70
5 Some Remarks Concerning Section 4,	71
6 Sketch of Another Approach,	72
7 Expected Sojourn,	73
8 A “Many-State” Case—Invariant Imbedding Approach,	74
9 Time Dependent Processes—Classical Approach,	76
10 Time-dependent Processes—Invariant Imbedding Approach,	77
11 A Multistep Process—Classical Approach,	78
12 A Multistep Process—Invariant Imbedding Approach,	79
13 Some Remarks on an Extension to a Continuous Case,	81
14 Some Remarks About Random Walk in Two Dimensions,	82
15 Summary,	83

- Problems, 83
- References, 87

Chapter 6 Wave Propagation**88**

- 1 Introduction, 88
- 2 The Concept of a Plane Wave, 89
- 3 A Two Medium Problem, 89
- 4 A Multimediu Problem, 90
- 5 Resolution of the Multimediu Problem by "Wavelet Counting", 91
- 6 A Continuous Medium Problem, 94
- 7 An Analytical Approach to the Continuous Medium Problem, 97
- 8 The W.K.B. Method, 98
- 9 The Bremmer Series, 100
- 10 Another Imbedding, 103
- 11 Summary, 105
- Problems, 105
- References, 107

Chapter 7 Time-Dependent Problems**108**

- 1 Introduction, 108
- 2 A Time-Dependent Transport Problem—Particle-Counting Approach, 109
- 3 Time-Dependent Transport by Transform Techniques, 114
- 4 A Critique of the Foregoing, 116
- 5 Time-Dependent Input, 118
- 6 The Time-Dependent Wave Equation, 120
- 7 The Diffusion Equation, 121
- 8 Some Comments on the Previous Section, 124
- 9 A Critique of Sections 7 and 8, 125
- 10 Another Diffusion Problem, 126
- 11 A Final Diffusion Problem, 127
- 12 Summary, 129
- Problems, 130
- References, 132

Chapter 8 The Calculation of Eigenvalues for Sturm-Liouville Type Systems**133**

- 1 Introduction, 133
- 2 Eigenlengths for Transport-like Equations in One Dimension, 134

- 3 The Calculation of Eigenlengths, 134
- 4 Some Generalizations, 137
- 5 Results for Sturm-Liouville Systems, 138
- 6 Connection with the Prüfer Transformation, 140
- 7 Some Numerical Examples, 142
- 8 Summary, 144
- Problems, 145
- References, 146

Chapter 9 Schrödinger-Like Equations

147

- 1 Introduction, 147
- 2 Formulation of the Phase Shift Problem, 148
- 3 A Representation of the Solution for Large t , 149
- 4 Partial Differential Equations for a and ψ , 151
- 5 Solution of the Partial Differential Equations for a and ψ , 153
- 6 Remarks on the Phase Shift Problem, 154
- 7 Formulation of the Eigenvalue Problem, 155
- 8 A Partial Differential Equation for \tilde{b} and Its “Solution”, 158
- 9 Resolution of the Difficulties, 161
- 10 Some Numerical Examples, 164
- 11 Some Remarks on the Eigenvalue Problem, 165
- 12 Summary, 166
- Problems, 167
- References, 168

Chapter 10 Applications to Equations with Periodic Coefficients

169

- 1 Introduction, 169
- 2 Statement of the Problem, 170
- 3 The Differential Equations of Invariant Imbedding Over One Period, 171
- 4 Difference Equations Over an Integral Number of Periods, 171
- 5 Difference Equations Over a Nonintegral Number of Periods, 176
- 6 The “Backwards” Equations, 177
- 7 Some Numerical Results, 178
- 8 The Method of Doubling, 180
- 9 Trigonometry Revisited, 181
- 10 Summary, 182

- Problems, 183
- References, 185

Chapter 11 Transport Theory and Radiative Transfer **186**

- 1 Introduction, 186
- 2 The Linearized Boltzmann Equation, 187
- 3 Some Remarks on Sections 1 and 2, 190
- 4 Boundary and Initial Conditions, 192
- 5 The Special Case of Slab Geometry and One Speed, 193
- 6 The Time-Independent Slab Problem via Invariant Imbedding—The Perturbation Approach, 196
- 7 The Time-Independent Slab Problem Via Invariant Imbedding—The Riccati Transformation, 202
- 8 A Return to the Case of the Semi-Infinite Half-Space, 206
- 9 Invariant Imbedding as a Computational Device for Transport Problems in a Slab, 208
- 10 Transport Theory in Other Geometries, 209
- 11 Time-Dependent Transport in a Slab Geometry, 210
- 12 Summary, 214
 - Problems, 215
 - References, 217

Chapter 12 Integral Equations **219**

- 1 Introduction, 219
- 2 An Integral Equation for Transport in a Slab, 220
- 3 A Pseudo-Transport Problem and Its Associated Integral Equation, 223
- 4 Representations for ϕ and n , 226
- 5 Derivation of the Principal Results, 227
- 6 A Special Case, 232
- 7 A Numerical Example and Some Remarks About Eigenvalues, 234
- 8 Further Remarks About the Foregoing, 238
- 9 A Completely Different Approach, 239
- 10 Summary, 242
 - Problems, 242
 - References, 245

Author Index **247**

Subject Index **249**