

CONTENTS

INTRODUCTION	xiii
1. INTEGRAL EQUATIONS	1
1.1. General remarks	2
1.2. Preliminary classification of linear integral equations	3
1.3. Linear equations of various types	6
1.4. The solvability of linear integral equations	10
1.5. Linear operators on a function space	17
1.6. Theoretical discussion of the eigenproblem	28
1.7. Equations of the second kind	33
1.8. Equations of the first kind	40
1.9. Elementary functional analysis	44
1.10. Differential equations and integral equations	56
1.11. Smoothness of kernels	68
1.12. Non-linear integral equations	77
2. NUMERICAL ANALYSIS	85
2.1. Introduction	86
2.2. Interpolation	87
2.3. Error in polynomial interpolation	92
2.4. Least-squares fitting	95
2.5. Fourier series	97
2.6. Error bounds	99
2.7. Generalized Fourier series	100
2.8. Closure and completeness	101
2.9. Discrete weighted least-squares approximations	103
2.10. Approximation in more than one dimension	104
2.11. Numerical integration	105
2.12. Repeated quadrature rules	109
2.13. Modifications of the trapezium rule	113
2.14. Classification of quadrature rules	123
2.15. Convergence	124

2.16.	Precise error bounds	128
2.17.	Integration of products	131
2.18.	Integration in more than one dimension	144
2.19.	Asymptotic upper and lower bounds	147
2.20.	Numerical methods	152
3.	EIGENVALUE PROBLEMS	167
3.1.	Introduction	168
3.2.	The quadrature method	169
3.3.	A modification of the quadrature method	193
3.4.	Product-integration methods	198
3.5.	Hämmerlin's approximate kernel method	202
3.6.	Expansion methods	205
3.7.	The collocation method	206
3.8.	The Galerkin method	214
3.9.	A least-squares method	219
3.10.	The Rayleigh quotient	220
3.11.	The Rayleigh-Ritz equations	222
3.12.	Further remarks	225
3.13.	Theoretical results	227
3.14.	The quadrature method for degenerate kernels	231
3.15.	The quadrature method for general kernels	234
3.16.	Error bounds for computed eigenvalues of Hermitian kernels	269
3.17.	A posteriori error bounds dependent on the local error	278
3.18.	A dual approach: Hermitian kernels	287
3.19.	The rôle of Peano theory	289
3.20.	Asymptotic error bounds for simple eigenvalues	293
3.21.	Asymptotic expansions for the error in a simple eigenvalue and the corresponding eigenfunction	297
3.22.	Extensions	314
3.23.	Expansion methods	316
3.24.	The Rayleigh-Ritz method for Hermitian kernels	316

3.25.	Error bounds for the Rayleigh-Ritz method for Hermitian kernels	322
3.26.	Convergence of the eigenvalues and eigenfunctions	325
3.27.	A further error bound	329
3.28.	The Rayleigh quotient	330
3.29.	Galerkin's method and the Rayleigh-Ritz method	330
3.30.	Convergence of the Galerkin method	336
3.31.	Convergence of the collocation method	340
3.32.	A product-integration method	346
3.33.	A generalized Rayleigh quotient and a least-squares solution	349
4.	LINEAR EQUATIONS OF THE SECOND KIND	352
4.1.	Introductory remarks	353
4.2.	The Fredholm equation of the second kind	353
4.3.	The quadrature method	356
4.4.	A modification to the quadrature method	375
4.5.	A further modification	379
4.6.	A product-integration method	380
4.7.	Reduction to a degenerate kernel	385
4.8.	Bateman's method	388
4.9.	Expansion methods and quadrature methods	389
4.10.	An infinite system of equations	391
4.11.	The collocation method	396
4.12.	Ritz-Galerkin type methods and minimization methods	406
4.13.	Linear programming methods	416
4.14.	Further remarks; conditioning	421
4.15.	The theory of methods for Fredholm equations of the second kind	423
4.16.	Analysis of the quadrature method	432
4.17.	The Anselone-Brakhage-Mysovskih approach	457
4.18.	Asymptotic expansions	466
4.19.	Deferred correction	473
4.20.	The modified quadrature method	477

4.21.	A product-integration method	479
4.22.	Expansion methods	481
4.23.	Variational formulation of the Rayleigh-Ritz method	482
4.24.	Collocation	487
4.25.	Ritz-Galerkin-type methods	496
4.26.	The classical Galerkin method	500
4.27.	The Galerkin methods using Chebyshev polynomials	505
4.28.	The method of least squares and the method of moments	507
4.29.	Projection methods	509
5.	FURTHER DISCUSSION OF THE TREATMENT OF FREDHOLM EQUATIONS	519
5.1.	Introduction	520
5.2.	Linear equations	521
5.3.	'Mild' and weak singularities in linear equations	526
5.4.	The modified quadrature method applied to equations with weakly singular kernels	534
5.5.	Simple product-integration formulae for weakly singular kernels	539
5.6.	Practical product-integration methods applied to weakly singular kernels	546
5.7.	Expansion methods for weakly singular kernels	556
5.8.	The eigenvalue problem for a weakly singular kernel	563
5.9.	Equations of the second kind which are not uniquely solvable	570
5.10.	Singular integral equations	577
5.11.	Theory of methods for continuous and weakly singular kernels	589
5.12.	An abstract theory	596
5.13.	Convergence of the modified quadrature method	603
5.14.	Convergence of the general product-integration method	616
5.15.	Fredholm equations of the first kind	635
5.16.	Quadrature methods and expansion methods	645

5.17.	A least-squares approach and regularization methods	655
5.18.	Non-linear integral equations	685
5.19.	The quadrature methods	686
5.20.	Expansion methods	699
5.21.	Collocation	701
5.22.	Galerkin methods	707
5.23.	Variational formulation	709
5.24.	Variational methods	717
5.25.	Theoretical study of methods for non-linear integral equations	719
6.	VOLTERRA INTEGRAL EQUATIONS	755
6.1.	Introduction	755
6.2.	The linear equation of the second kind and Volterra type	759
6.3.	Stability and ill-conditioning	788
6.4.	Quadrature methods for non-linear integral equations	825
6.5.	Starting values	835
6.6.	Runge-Kutta-type methods for Volterra equations	849
6.7.	A block-by-block method	864
6.8.	Product-integration methods	885
6.9.	Product-integration techniques for singular linear and non-linear equations	892
6.10.	Non-singular Volterra equations of the first kind	896
6.11.	Product-integration methods for equations of the first kind with continuous or weakly singular kernels	915
6.12.	Convergence properties of numerical methods for equations of the second kind	922
6.13.	Convergence of certain block-by-block methods	952
6.14.	Convergence of product-integration methods	959
6.15.	Convergence of methods for non-singular equations of the first kind	963

6.16.	Block-by-block methods for non-singular equations of the first kind	974
6.17.	Theory of product-integration methods for first kind equations	979
REFERENCES		985
INDEX		1031

