TABLE OF CONTENTS

INTRODUCTION 1

ACKNOWLEDGEMENTS 2

Part I. BASIC RESULTS OF FUNCTIONAL ANALYSIS . . 3

CHAPTER

1.0 Vector spaces 3

2.0 Operators on vector spaces 8

3.0 The geometric series and its application 13

Approximation of linear operator equations 15

4.0 Principle of uniform boundedness 18

Application to numerical integration 20

5.0 Compact integral operators 23

Integral operators on C[a,b] 24

The Fredholm alternative theorem 26

Summary of other theoretical results 29

Part II. NUMERICAL METHODS FOR COMPACT INTEGRAL EQUATIONS OF THE SECOND KIND . 32

1.0 Successive approximations 32

2.0 Degenerate kernel and projection methods 37

2.1 Degenerate kernel methods 37

Solution of degenerate kernel equation 39

Forming degenerate kernel approximations 42

Orthonormal expansions 46

Interpolatory approximations 47

TABLE OF CONTENTS (Continued)

CHAPTER

2.2 Projection methods 50

Collocation method 54

Numerical examples 58

Galerkin method 62

Regularizing $(\lambda - \mathcal{K})x = y$ 71

2.3 Stability of linear systems 73

Collocation method 76

Galerkin method 79

Discussion of literature 84

3.0 The Nyström method 88

3.1 An abstract error analysis 93

Perturbation theorem 94

Collectively compact operator theory 96

Application of abstract error analysis 99

Numerical examples 102

Stability of linear system 105

3.2 Singular kernels and product integration 106

Product integration method 1 106

Error analysis 108

Calculation of weights 113

Numerical examples 114

Product integration method 2 117

Numerical examples 119

Error analysis 122

TABLE OF CONTENTS (Continued)

CHAPTER

3.3 Connections with the collocation method 123

Discussion of literature 125

4.0 Iterative variants of direct methods 127

4.1 An iterative variant of the degenerate kernel method . . . 128

Numerical example 132

4.2 Iterative variants of the Nyström method 138

Method 1 139

Method 2 142

Method 3 147

Numerical examples 152

Discussion of literature 162

5.0 Automatic algorithms 164

5.1 An Automatic program based on Simpson's rule 165

Numerical examples 171

5.2 An automatic program based on Gaussian quadrature . . 175

Numerical examples 179

5.3 The computer programs 187

Flowchart of IESIMP 189

Explanation of program variable names 191

Program IESIMP 192

Program IEGAUS 204

References 223