CONTENTS

Chapter VI. THE SPECTRAL ANALYSIS OF UNITARY AND SELF-ADJOINT OPERATORS 1

58. The Trigonometric Moment Problem 1
59. Analytic Functions with Values in a Half-plane 5
60. The Theorem of Bochner 11
61. The Resolution of the Identity 14
62. The Integral Representation of a Unitary Operator 16
63. Operators Represented by Stieltjes Integrals 22
64. The Integral Representation of a Group of Unitary Operators 29
65. The Integral Representation of the Resolvent of a Self-Adjoint Operator 31
66. The Integral Representation of Self-Adjoint Operators 36
67. The Cayley Transform 42
68. The Spectra of Self-Adjoint and Unitary Operators 46
69. The Simple Spectrum 50
70. Spectral Types 56
71. The Multiple Spectrum 59
72. The Canonical Form of a Self-Adjoint Operator with a Spectrum of Finite Multiplicity 60
73. Some Remarks about Unitary Invariants of Self-Adjoint Operators 65
74. Some Remarks about Functions of Self-Adjoint Operators 74
75. Commutative Operators 76
76. Rings of Bounded Self-Adjoint Operators 80
77. Examples 84

Chapter VII. THEORY OF EXTENSIONS OF SYMMETRIC OPERATORS 91

78. Deficiency Indices 91
79. Further Remarks on the Cayley Transform 94
80. The Neumann Formulas 97
81. Simple Symmetric Operators 101
82. The Structure of Maximal Operators 103
83. Spectra of Self-Adjoint Extensions of Symmetric Operators 107
84. The Formula of Krein for the Resolvent of the Self-Adjoint Extensions of a Symmetric Operator 110
85. Semi-Bounded Operators 114
86. Some Remarks about the General Theory of Extensions 119

Appendix I. GENERALIZED EXTENSIONS AND GENERALIZED SPECTRAL FUNCTIONS OF SYMMETRIC OPERATORS 121

1. Generalized Resolution of the Identity. Naimark's Theorem 121
2. Self-Adjoint Extensions to Larger Spaces and Spectral Functions of Symmetric Operators 126
3. Spectral Functions of Symmetric Operators and Generalized Resolvents 133
4. The Formula of Krein for Generalized Resolvents 139
5. Quasi-Self-Adjoint Extensions and the Characteristic Function of a Symmetric Operator 146

Appendix II. DIFFERENTIAL OPERATORS 162

1. Self-Adjoint Differential Expressions 162
2. Regular Differential Operators 166
3. Self-Adjoint Extensions of a Regular Differential Operator 168
4. Singular Differential Operators 170
5. Self-Adjoint Extensions of a Singular Differential Operator 174
6. The Resolvents of Self-Adjoint Extensions 177
7. Inversion Formulas Related to Differential Operators of the Second Order 186
8. Generalization to Differential Operators of Arbitrary Order 200
9. Examples 204

INDEX 216