

Contents

<i>Preface</i>	xi
<i>Acknowledgments</i>	xvii

Chapter 0 Algebraic Preliminaries

1	Conventions	1
2	Separable dependence	2
3	Quasi-separable field extensions	4
4	Quotients	7
5	Perfect ideals	7
6	Separable, quasi-separable, and regular ideals	8
7	Conservative systems	10
8	Perfect conservative systems	12
9	Noetherian conservative systems	13
10	Morphisms and birational equivalence of ideals	16
11	Polynomial ideals and generic zeros	19
12	Polynomial ideals and ground field extension	20
13	Power series	29
14	Specializations	33
15	Algebraic function fields of one variable	41
16	Dimension of components	43
17	Lattice points	49
18	Shapiro's lemma	53
19	\mathfrak{f} -Values	56

Chapter I Basic Notions of Differential Algebra

1	Differential rings	58
2	Homomorphisms and differential ideals	61
3	Differential rings of quotients	63
4	Transformation and restriction of the set of derivation operators	65
5	Differential modules; differential algebras	66
6	Differential polynomial algebras	69
7	Permissible gradings	72
8	Rank	75
9	Autoreduced sets	77
10	Characteristic sets	81
11	Pseudo-leaders	83
12	Differential algebras of power series	84

Chapter II Differential Fields

1	Linear dependence over constants	86
2	Separable extensions	89
3	Differentially perfect and differentially quasi-perfect differential fields	92
4	Separable dependence over constants	93
5	Differential polynomial functions	95
6	Dependence of derivative operators	95
7	Differentially separable dependence	99
8	Differentially separable extensions	100
9	Differential inseparability bases	104
10	Differential transcendence bases	108
11	Finitely generated extensions	109
12	Differential inseparability polynomials	115
13	Differential type; typical differential inseparability degree	118

Chapter III The Basis Theorem and Some Related Topics

1	Differential conservative systems	121
2	Quasi-separable differential ideals	123
3	Differential fields of definition	125
4	The basis theorem	126
5	Differential dimension polynomials	129
6	Extension of the differential field of coefficients	130
7	Universal extensions	133
8	\mathfrak{f} -Coherent autoreduced sets	135
9	Differential specializations	138
10	Constrained families	142

Chapter IV Algebraic Differential Equations**PART A. CHARACTERISTIC p ARBITRARY**

1	Differential affine space. The differential Zariski topology	145
2	Generic zeros. The theorem of zeros	146
3	Closed sets and \mathcal{U} -separable differential ideals	147
4	The relative topologies; differential fields of definition	148
5	Linear differential ideals	150
6	General components	155
7	General components and differential dimension polynomials	160
8	Multiplicity of zeros	164

PART B. CHARACTERISTIC $p = 0$

9	Finite sets of differential polynomials	166
10	The leading coefficient theorem	171
11	Levi's lemma	176
12	The domination lemma	178
13	Preparations	183
14	The component theorem	185
15	The low power theorem	187
16	The Ritt problem	190
17	Systems of bounded order	194
18	Substitution of powers	202

Bibliography for Chapters I–IV	206
---------------------------------------	------------

Chapter V Algebraic Groups

1	Introduction	212
2	Pre- K -sets	215
3	K -Groups and homogeneous K -spaces. K -Sets	218
4	Extending the universal field	227
5	Extending the basic field	230
6	Zariski topology; K -topology	236
7	Closed sets	240
8	K -Subgroups	247
9	K -Homomorphisms	249
10	Direct products	257
11	Quotients	267
12	Galois cohomology	273
13	Principal homogeneous K -spaces	281
14	Holomorphicity at a specialization	287
15	K -Mappings	294
16	K -Functions	306
17	K -Cohomology	318
18	Invariant derivations and differentials. The Lie algebra	322

19	Local rings	331
20	Tangent spaces	334
21	Crossed K -homomorphisms	341
22	Logarithmic derivatives	349
23	Linear K -groups	354
24	Abelian K -groups	376

Bibliography for Chapter V	383
-----------------------------------	-----

Chapter VI Galois Theory of Differential Fields

1	Specializations of isomorphisms	385
2	Strong isomorphisms	388
3	Strongly normal extensions. Galois groups	393
4	The fundamental theorems	398
5	Examples	404
6	Picard–Vessiot extensions	409
7	G -Primitives	418
8	Differential Galois cohomology	421
9	Applications	426
10	V -Primitives	427

Bibliography for Chapter VI	431
------------------------------------	-----

<i>Glossary of Notation</i>	435
<i>Index of Definitions</i>	441

