Contents

Preface v
Acknowledgements x
List of Symbols xv

1. Introduction 1
1.1 Differential input-output relations and systems 4
1.2 Compositions of differential input-output relations 7
1.3 Algebraic foundations 13
1.4 Properties of differential input-output relations 22
1.5 Feedback compensation and control 27
1.6 Concluding remarks 44

Part I Basic Concepts of Systems Theory

2. Systems and system descriptions 49
2.1 Parametric input-output mappings. Abstract input-output systems. System descriptions 50
2.2 Time systems. Dynamic systems 52
2.3 Linear systems 53

3. Interconnections of systems 56
3.1 Formal definition of an interconnection of a family of systems . . 56
3.2 Input-output relations determined by an interconnection of a family of systems 61
3.3 Determinateness with respect to the empty set of realizability conditions 63
3.4 Determinateness with respect to a general set of realizability conditions 64
3.5 Illustrative example 67

Part II Differential Systems. The Module Structure

4. Generation of differential systems 75
4.1 Signal spaces and differential operators 75
4.2 Matrix differential equations 77

5. The $\mathbf{C}[p]$*-module* $\mathscr{X}$ 80
5.1 Suitable signal spaces $\mathscr{X}$ 81
5.2 The ring $\mathbf{C}[p]$ of polynomial operators. The $\mathbf{C}[p]$-module $\mathscr{X}$ . . 84
5.3 Relationship between polynomial matrix operators and matrices over $\mathbf{C}[p]$ 85
5.4 Fundamental properties of polynomial matrix operators . . . 86

6. Differential input-output relations. Generators 88
6.1 Introduction. Regular differential input-output relations and regular generators 89
6.2 Input-output equivalence. Complete invariants and canonical forms for input-output equivalence 91
6.3 The transfer matrix. Proper and strictly proper transfer matrices, generators, and differential input-output relations 94
6.4 Transfer equivalence. Complete invariants and canonical forms for transfer equivalence. Controllability 95
6.5 Proofs of theorems (6.2.1), (6.2.2) and (6.4.8) 100
6.6 Comments on canonical forms. Canonical row proper forms . . 104

7. Analysis and synthesis problems 106
7.1 An elimination procedure 107
7.2 Compositions and decompositions of regular differential input-output relations. Observability 110
7.3 A parallel composition 123
7.4 Parallel decompositions of regular differential input-output relations 126
7.5 Illustrative example 133
7.6 A series composition 137
7.7 Series and series-parallel decompositions of regular differential input-output relations 143
7.8 The Rosenbrock representation 145
7.9 The state-space representation 158
7.10 The Rosenbrock representation and the state-space representation as decompositions of regular differential input-output relations. Equivalence relations 160
7.11 Observer synthesis problem 176
7.12 Feedback compensator synthesis 184

Part III Differential Systems. The Vector Space Structure

8. The projection method 203
8.1 Reason for choosing a space of generalized functions as signal space 204
8.2 The basic signal space $\mathscr{D}$ of generalized functions. Projection mappings. Subspaces of $\mathscr{D}$. Generalized causality 205
8.3 The vector space $\mathscr{X}$ over $\mathbf{C}(p)$ 210
8.4 Compositions of projections and differential operators. Initial condition mappings 216
8.5 The projection method 226

9. Interconnections of differential systems 243
9.1 Two interconnections 243
9.2 Systems associated with compositions of input-output relations . . 250

9.3 The main results 253
9.4 Illustrative example 259

Part IV Difference Systems

10. Generation of difference systems 273
10.1 Signal spaces and shift operators 273
10.2 Matrix difference equations 274

11. The module structure 276
11.1 Suitable signal spaces 276
11.2 The rings $\mathbf{C}[z]$, $\mathbf{C}(z]$, $\mathbf{C}[1/z]$ and $\mathbf{C}(z)$ and modules over them . . 278
11.3 Polynomial and rational matrix operators and polynomial and rational matrices 279

12. Difference input-output relations. Generators 281
12.1 Regular difference input-output relations and regular generators . 282
12.2 Input-output equivalence. Canonical forms for input-output equivalence. Causality 283
12.3 The transfer matrix. Properness and causality 284
12.4 Transfer equivalence. Canonical forms for transfer equivalence Controllability. 285

13. Analysis and synthesis problems 288
13.1 Compositions and decompositions of regular difference input-output relations. Observability 288
13.2 The feedback composition 291

14. The vector space structure. The projection method 293
14.1 Signal space 293
14.2 The modules and vector spaces of quotients 295
14.3 Compositions of projections and delay operators. Initial condition mappings 296
14.4 The projection method 298

Appendices

A1 Fundamentals of abstract algebra 303

A2 Polynomials and polynomial matrices 321

A3 Polynomials and rational forms in an endomorphism . . . 340

A4 The space $\mathcal{D}$ of generalized functions 342

References 349
Index 353