

Contents of Volume 1

Contents of Volume 2	x
Preface	xiii
Organization of the Book	xvii
Acknowledgments	xxi
Introduction	xxiii
PART ONE: THE RAW MATERIALS OF MATHEMATICS	1
Chapter 1 Sets	3
1.1 Operations with Sets	6
1.2 Relations in Sets	13
1.2a. Equivalence relations	15
1.2b. Order relations	20
Chapter 2 Maps	26
2.1 Composite Functions and Inverses	32
2.2 Equivalence Relations and Maps	38
2.3 Ordered Sets and Maps	42
2.4 Cardinal Numbers	43
2.5 Sequences and Families	47
	vii

PART TWO: THE BASIC STRUCTURES OF MATHEMATICS	53
<i>IIA: Algebraic Structures</i>	
Chapter 3 Algebraic Composition Laws and Systems	57
3.1 Morphisms of Algebraic Systems	62
Chapter 4 Survey of Special Algebraic Systems	68
4.1 Groups	70
4.1a. Transformation groups; G -spaces; orbits	79
4.1b. Conjugate classes; cosets	87
4.1c. Normal subgroups; quotient groups; isomorphism theorems	91
4.2 Rings and Fields	102
4.2a. Ideals; quotient rings; isomorphism theorems	115
4.3 Linear Spaces	120
4.3a. Linear independence, bases and dimension	128
4.3b. Morphisms (linear transformations); quotient spaces	137
4.4 Linear Algebras	149
4.4a. Morphisms of algebras; quotient algebras	159
4.5 Nonassociative Algebras	167
4.5a. Lie algebras	168
4.5b. Some other nonassociative algebras	182
<i>IIB: Topological Structures</i>	
Chapter 5 Topological Spaces	187
5.1 Examples; Metric Spaces	188
5.2 General Structure of Topological Spaces	198
5.3 Neighborhoods; Special Points; Closed Sets	203
5.3a. Interior, closure, boundary	208
5.4 Convergence	211
5.5 Continuity	216
5.6 Homeomorphism and Isometry	220
5.6a. Quotient topology; homeomorphism theorem	229
Chapter 6 Topological Spaces with Special Properties	236
6.1 Connected Spaces	236
6.1a. Path connectivity; homotopy	242
6.2 Separable Spaces	250

6.3	Compact Spaces	254
6.3a.	Compactification	266
6.4	Complete Metric Spaces	270
6.4a.	Completion	275
6.4b.	Contraction mappings	280
	<i>IIC: Measure Structures</i>	291
Chapter 7	Measure Spaces	293
7.1	Measurable Spaces	294
7.2	Measure and Measure Spaces	305
7.2a.	General properties of measures	310
7.2b.	Lebesgue measure	314
7.2c.	Lebesgue–Stieltjes measures	320
7.2d.	Signed and complex measure	325
Chapter 8	Theory of Integration	328
8.1	Measurable Functions	329
8.2	Definition of the Integral	338
8.3	General Properties of the Integral	353
8.4	Comments on Lebesgue and Lebesgue–Stieltjes Integrals	365
8.5	The Radon–Nikodym Theorem	371
	APPENDICES	xxvii
Appendix I	Some Inequalities	xxix
Appendix III	Annotated Reading List	xxx
Appendix IV	Frequently Used Symbols	xxxvi
Index		xliii