## **CONTENTS**

EDITOR'S FOREWORD	`
PREFACE TO THE FIRST EDITION	vii
PREFACE TO THE THIRD EDITION	vii
PREFACE TO THE FOURTH EDITION	viii
THE ORDER OF PRESENTATION OF THE FORMULAS	xxiii
USE OF THE TABLES	xxvii
INDEX OF SPECIAL FUNCTIONS AND NOTATIONS	xxxix
NOTATIONS	xliii
NOTE ON THE BIBLIOGRAPHIC REFERENCES	xlv
0. INTRODUCTION	1
0.1 Finite Sums	1
0.11 Progressions	1
0.12 Sums of powers of natural numbers	1
0.13 Sums of reciprocals of natural numbers	2
0.14 Sums of products of reciprocals of natural numbers	3
0.15 Sums of the binomial coefficients	3
0.2 Numerical Series and Infinite Products	5
0.21 The convergence of numerical series	5
0.22 Convergence tests	5
0.23-0.24 Examples of numerical series	7
0.25 Infinite products	11
0.26 Examples of infinite products	12
0.3 Functional Series	12
0.30 Definitions and theorems	12
0.31 Power series	13
0.32 Fourier series	16
0.33 Asymptotic series	18
0.4 Certain Formulas from Differential Calculus	18
0.41 Differentiation of a definite integral with respect to a param-	
eter	18
0.42 The nth derivative of a product	10

	0.43 The nth derivative of a composite function	19
l.	ELEMENTARY FUNCTIONS	21
	1.1 Power of Binomials	21
	1.11 Power series	21
	1.12 Series of rational fractions	22
	1.2 The Exponential Function	22
	1.21 Series representations	22
	1.22 Functional relations	23
	1.23 Series of exponentials	23
	1.3-1.4 Trigonometric and Hyperbolic Functions	23
	1.30 Introduction	23
	1.31 The basic functional relations	24
	1.32 The representation of powers of trigonometric and hyperbolic	
	functions in terms of functions of multiples of the argument (angle)	25
	1.33 The representation of trigonometric and hyperbolic functions	
	of multiples of the argument (angle) in terms of powers of these	
	functions	27
	1.34 Certain sums of trigonometric and hyperbolic functions	29
	1.35 Sums of powers of trigonometric functions of multiple angles	30
	1.36 Sums of products of trigonometric functions of multiple angles	32
	1.37 Sums of tangents of multiple angles	32
	1.38 Sums leading to hyperbolic tangents and cotangents	32
	1.39 The representation of cosines and sines of multiples of the	
	angle as finite products	33
	1.41 The expansion of trigonometric and hyperbolic functions in	
	power series	34
	1.42 Expansion in series of simple fractions	36
	1.43 Representation in the form of an infinite product	37
	1.44-1.45 Trigonometric (Fourier) series	38
	1.46-Series of products of exponential and trigonometric functions	42
	1.47 Series of hyperbolic functions	42
	1.48 Lobachevskiy's "Angle of parallelism" $\Pi(x)$	43
	1.49 The hyperbolic amplitude (the Gudermannian) $\operatorname{gd} x$	43
	1.5 The Logarithm	44
	1.51 Series representation	44
	1.52 Series of logarithms	<b>4</b> 6
	1.6 The Inverse Trigonometric and Hyperbolic Functions	47
	1.61 The domain of definition	47
	1.62-1.63 Functional relations	47
	1.64 Series representations	51

CONTENTS	xi
. INDEFINITE INTEGRALS OF ELEMENTARY FUNCTIONS	53
2.0 Introduction	53
2.00 General remarks	53
2.01 The basic integrals	54
2.02 General formulas	55
2.1 Rational Functions	56
2.10 General integration rules	56
2.11-2.13 Forms containing the binomial $a + bx^k$	58
2.14 Forms containing the binomial $1 \pm x^n$	63
2.15 Forms containing pairs of binomials: $a + bx$ and $a + \beta x$	66
2.16 Forms containing the trinomial $a + bx^k + cx^{2k}$	67
2.17 Forms containing the quadratic trinomial $a + bx + cx^2$ and	
powers of x	68
2.18 Forms containing the quadratic trinomial $a + bx + cx^2$ and	
the binomial $\alpha + \beta x$	70
2.2 Algebraic Functions	70
2.20 Introduction	70
2.21 Forms containing the binomial $a + bx^k$ and $\sqrt{x}$	71
2.22-2.23 Forms containing $\sqrt[n]{(a+bx)^k}$	72
2.24 Forms containing $\sqrt{a + bx}$ and the binomial $a + \beta x$	75
2.25 Forms containing $\sqrt{a + bx + cx^2}$	80
2.26 Forms containing $\sqrt{a + bx + cx^2}$ and integral powers of x	81
2.27 Forms containing $\sqrt{a + cx^2}$ and integral powers of x	86 ************************************
2.28 Forms containing $\sqrt{a + bx + cx^2}$ and first- and second-deg	ree 89
polynomials 2.29 Integrals that can be reduced to elliptic or pseudo-elliptic	
integrals	90
2.3 The Exponential Function	92
2.31 Forms containing $e^{ax}$	92
2.32 The exponential combined with rational functions of x	92
2.4 Hyperbolic Functions	93
2.41-2.43 Powers of $\operatorname{sh} x$ , $\operatorname{ch} x$ , $\operatorname{th} x$ and $\operatorname{cth} x$	93
2.44-2.45 Rational functions of hyperbolic functions	107
2.46 Algebraic functions of hyperbolic functions	113
2.47 Combinations of hyperbolic functions and powers	119
2.48 Combinations of hyperbolic functions, exponentials, and	
powers	128
2.5-2.6 Trigonometric Functions	129
2.50 Introduction	129
2.51-2.52 Powers of trigonometric functions	130

2.53-2.54 Sines and cosines of multiple angles and of linear and	
more complicated functions of the argument	139
2.55-2.56 Rational functions of the sine and cosine	147
2.57 Forms containing $\sqrt{a \pm b \sin x}$ , or $\sqrt{a \pm b \cos x}$ and forms re-	
ducible to such expressions	153
2.58-2.62 Integrals reducible to elliptic and pseudo-elliptic	
integrals	157
2.63-2.65 Products of trigonometric functions and powers	182
2.66 Combinations of trigonometric functions and exponentials	194
2.67 Combinations of trigonometric and hyperbolic functions	198
2.7 Logarithms and Inverse-Hyperbolic Functions	203
2.71 The logarithm	203
2.72-2.73 Combinations of logarithms and algebraic functions	203
2.74 Inverse hyperbolic functions	206
2.8 Inverse Trigonometric Functions	207
2.81 Arcsines and arccosines	207
2.82 The arcsecant, the arccosecant, the arctangent and the arccosecant	207
2.83 Combinations of arcsine or arccosine and algebraic functions	208
2.84 Combinations of the arcsecant and arccosecant with powers	-00
of $x$	209
2.85 Combinations of the arctangent and arccotangent with alge-	
braic functions	209
B 4. DEFINITE INTEGRALS OF ELEMENTARY FUNCTIONS	211
3.0 Introduction	211
3.01 Theorems of a general nature	211
3.02 Change of variable in a definite integral	212
3.03 General formulas	213
3.04 Improper integrals	215
3.05 The principal values of improper integrals	216
3.1-3.2 Power and Algebraic Functions	217
3.11 Rational functions	217
3.12 Products of rational functions and expressions that can be	
reduced to square roots of first- and second-degree poly-	
nomials	219
3.13-3.17 Expressions that can be reduced to square roots of	
third- and fourth-degree polynomials and their products with	A3.0
rational functions	219
3.18 Expressions that can be reduced to fourth roots of second-	22.7
degree polynomials and their products with rational functions	282

CONTENTS	xiii
3.19-3.23 Combinations of powers of x and powers of binomials of	
the form $(\alpha + \beta x)$	284
3.24-3.27 Powers of x, of binomials of the form $a + \beta x^p$ and of	
polynomials in $x$	292
3.3-3.4 Exponential Functions	304
3.31 Exponential functions	304
3.32-3.34 Exponentials of more complicated arguments	306
3.35 Combinations of exponentials and rational functions	310
3.36-3.37 Combinations of exponentials and algebraic functions	315
3.38-3.39 Combinations of exponentials and arbitrary powers	317
3.41-3.44 Combinations of rational functions of powers and expo-	
nentials	325
3.45 Combinations of powers and algebraic functions of exponen-	
tials	335
3.46-3.48 Combinations of exponentials of more complicated argu-	
ments and powers	337
3.5 Hyperbolic Functions	344
3.51 Hyperbolic functions	344
3.52-3.53 Combinations of hyperbolic functions and algebraic	
functions	347
3.54 Combinations of hyperbolic functions and exponentials	356
3.55-3.56 Combinations of hyperbolic functions, exponentials and	
powers	360
3.6-4.1 Trigonometric Functions	365
3.61 Rational functions of sines and cosines and trigonometric	
functions of multiple angles	365
3.62 Powers of trigonometric functions	369
3.63 Powers of trigonometric functions and trigonometric functions	0.00
of linear functions	372
3.64-3.65 Powers and rational functions of trigonometric functions	377
3.67 Square roots of expressions containing trigonometric functions	386
3.68 Various forms of powers of trigonometric functions	389
3.69-3.71 Trigonometric functions of more complicated arguments	395
3.72-3.74 Combinations of trigonometric and rational functions 3.75 Combinations of trigonometric and algebraic functions	405
3.76-3.77 Combinations of trigonometric functions and powers	418
3.78-3.81 Rational functions of $x$ and of trigonometric functions	420
3.82-3.83 Powers of trigonometric functions combined with other	432
powers	A A C
3.84 Integrals containing the expressions $\sqrt{1-k^2\sin^2 x}$ ,	446
$\sqrt{1-k^2\cos^2x}$ and similar expressions	460
12 " COO " WITH DIMITION CAPICODIONIS	400

xiv CONTENTS

3.85-3.88 Trigonometric functions of more complicated arguments	
combined with powers	464
3.89-3.91 Trigonometric functions and exponentials	476
3.92 Trigonometric functions of more complicated arguments com-	
bined with exponentials	484
3.93 Trigonometric and exponential functions of trigonometric	
functions	486
3.94-3.97 Combinations involving trigonometric functions, expo-	
nentials, and powers	489
3.98-3.99 Combinations of trigonometric and hyperbolic functions	503
4.11-4.12 Combinations involving trigonometric and hyperbolic	
functions and powers	511
4.13 Combinations of trigonometric and hyperbolic functions and	
exponentials	519
4.14 Combinations of trigonometric and hyperbolic functions, ex-	
ponentials, and powers	521
4.2-4.4 Logarithmic Functions	523
4.21 Logarithmic functions	523
4.22 Logarithms of more complicated arguments	525
4.23 Combinations of logarithms and rational functions	532
4.24 Combinations of logarithms and algebraic functions	535
4.25 Combinations of logarithms and powers	537
4.26-4.27 Combinations involving powers of the logarithm and	
other powers	539
4.28 Combinations of rational functions of ln x and powers	552
4.29-4.32 Combinations of logarithmic functions of more compli-	
cated arguments and powers	555
4.33-4.34 Combinations of logarithms and exponentials	573
4.35-4.36 Combinations of logarithms, exponentials, and powers	575
4.37 Combinations of logarithms and hyperbolic functions	580
4.38-4.41 Logarithms and trigonometric functions	583
4.42-4.43 Combinations of logarithms, trigonometric functions, and powers	598
4.44 Combinations of logarithms, trigonometric functions, and ex-	
ponentials	605
4.5 Inverse Trigonometric Functions	605
4.51 Inverse trigonometric functions	605
4.52 Combinations of arcsines, arccosines, and powers	606
4.53-4.54 Combinations of arctangents, arccotangents, and powers	607
4.55 Combinations of inverse trigonometric functions and expo-	
nentials	611

CONTENTS	,	ΧV
	4	A.V

	4.56 A combination of the arctangent and an inverse hyperbolic function	<i>(</i> 10
		612
	4.57 Combinations of inverse and direct trigonometric functions	612
	4.58 A combination involving an inverse and a direct trigonometric	(1/
	function and a power	614
	4.59 Combinations of inverse trigonometric functions and loga-	
	rithms	614
	4.6 Multiple Integrals	<b>61</b> 4
	4.60 Change of variables in multiple integrals	<b>61</b> 4
	4.61 Change of the order of integration and change of variables	615
	4.62 Double and triple integrals with constant limits	618
	4.63-4.64 Multiple integrals	620
5	. INDEFINITE INTEGRALS OF SPECIAL FUNCTIONS	626
	5.1 Elliptic Integrals and Functions	626
	5.11 Complete elliptic integrals	626
	5.12 Elliptic integrals	627
	5.13 Jacobian elliptic functions	629
	5.14 Weierstrass elliptic functions	631
	5.2 The Exponential-Integral Function	632
	5.21 The exponential-integral function	632
	5.22 Combinations of the exponential-integral function and powers	632
	5.23 Combinations of the exponential-integral and the exponential	632
	5.3 The Sine-Integral and the Cosine-Integral	632
	5.4 The Probability Integral and Fresnel Integrals	633
	5.5 Bessel Functions	633
6.	7. DEFINITE INTEGRALS OF SPECIAL FUNCTIONS	635
	6.1 Elliptic Integrals and Functions	635
	6.11 Forms containing $F(x, k)$	635
	6.12 Forms containing $E(x, k)$	636
	6.13 Integration of elliptic integrals with respect to the modulus	636
	6.14-6.15 Complete elliptic integrals	637
	6.16 The theta function	638
	6.2-6.3 The Exponential-Integral Function and Functions Generated	
	by It	639
	6.21 The logarithm-integral	639
	6.22-6.23 The exponential-integral function	641
	6.24-6.26 The sine- and cosine-integral functions	643
	6.27 The hyperbolic-sine- and -cosine-integral functions	647
	6.28-6.31 The probability integral	648
	<u>-</u>	

6.32 Fresnel integrals	653
6.4 The Gamma Function and Functions Generated by It	655
6.41 The gamma function	655
6.42 Combinations of the gamma function, the exponential, and	
powers	656
6.43 Combinations of the gamma function and trigonometric	
functions	660
6.44 The logarithm of the gamma function	661
6.45 The incomplete gamma function	662
6.46-6.47 The function $\psi(x)$	664
6.5-6.7 Bessel Functions	665
6.51 Bessel functions	665
6.52 Bessel functions combined with $x$ and $x^2$	672
6.53-6.54 Combinations of Bessel functions and rational functions	677
6.55 Combinations of Bessel functions and algebraic functions	681
6.56-6.58 Combinations of Bessel functions and powers	683
6.59 Combinations of powers and Bessel functions of more com-	
plicated arguments	700
6.61 Combinations of Bessel functions and exponentials	707
6.62-6.63 Combinations of Bessel functions, exponentials, and	
powers	711
6.64 Combinations of Bessel functions of more complicated argu-	
ments, exponentials, and powers	720
6.65 Combinations of Bessel and exponential functions of more	
complicated arguments and powers	723
6.66 Combinations of Bessel, hyperbolic, and exponential	
functions	726
6.67-6.68 Combinations of Bessel and trigonometric functions	730
6.69-6.74 Combinations of Bessel and trigonometric functions and	
powers	743
6.75 Combinations of Bessel, trigonometric, and exponential func-	
tions and powers	762
6.76 Combinations of Bessel, trigonometric, and hyperbolic func-	
tions	767
6.77 Combinations of Bessel functions and the logarithm, or arc-	
tangent	767
6.78 Combinations of Bessel and other special functions	768
6.79 Integration of Bessel functions with respect to the order	770
6.8 Functions Generated by Bessel Functions	775
6.81 Struve functions	775
6.82 Combinations of Struve functions, exponentials, and powers	777

CONTENTS	xvii
6.83 Combinations of Struve and trigonometric functions	778
6.84-6.85 Combinations of Struve and Bessel functions	779
6.86 Lommel functions	784
6.87 Thomson functions	787
6.9 Mathieu Functions	788
6.91 Mathieu functions	788
6.92 Combinations of Mathieu, hyperbolic, and trigonometric	
functions	789
6.93 Combinations of Mathieu and Bessel functions	793
7.1-7.2 Associated Legendre Functions	794
7.11 Associated Legendre functions	794
7.12-7.13 Combinations of associated Legendre functions and	
powers	795
7.14 Combinations of associated Legendre functions, exponentials,	
and powers	803
7.15 Combinations of associated Legendre and hyperbolic functions	806
7.16 Combinations of associated Legendre functions, powers, and	
trigonometric functions	806
7.17 A combination of an associated Legendre function and the	
probability integral	810
7.18 Combinations of associated Legendre and Bessel functions	810
7.19 Combinations of associated Legendre functions and functions	
generated by Bessel functions	817
7.21 Integration of associated Legendre functions with respect to	
the order	819
7.22 Combinations of Legendre polynomials, rational functions,	
and algebraic functions	820
7.23 Combinations of Legendre polynomials and powers	822
7.24 Combinations of Legendre polynomials and other elementary	
functions	823
7.25 Combinations of Legendre polynomials and Bessel functions	825
7.3-7.4 Orthogonal Polynomials	826
7.31 Combinations of Gegenbauer polynomials $C_n^v(x)$ and powers	826
7.32 Combinations of the polynomials $C_n^{\upsilon}(x)$ and some elementary	
functions	830
7.33 Combinations of the polynomials $C_n^{\upsilon}(x)$ and Bessel functions.	
Integration of Gegenbauer functions with respect to the index	831
7.34 Combinations of Chebyshev polynomials and powers	833
7.35 Combinations of Chebyshev polynomials and some elementary	
functions	835
7.36 Combinations of Chebyshev polynomials and Bessel functions	836

•

xviii CONTENTS

7.37-7.38 Hermite polynomials	836
7.39 Jacobi polynomials	841
7.41-7.42 Laguerre polynomials	843
7.5 Hypergeometric Functions	848
7.51 Combinations of hypergeometric functions and powers	848
7.52 Combinations of hypergeometric functions and exponentials	850
7.53 Hypergeometric and trigonometric functions	853
7.54 Combinations of hypergeometric and Bessel functions	853
7.6 Degenerate Hypergeometric Functions	857
7.61 Combinations of degenerate hypergeometric functions and	
powers	857
7.62-7.63 Combinations of degenerate hypergeometric functions	
and exponentials	859
7.64 Combinations of degenerate hypergeometric and trigonometric	
functions	869
7.65 Combinations of degenerate hypergeometric functions and	
Bessel functions	870
7.66 Combinations of degenerate hypergeometric functions, Bessel	
functions, and powers	871
7.67 Combinations of degenerate hypergeometric functions, Bessel	
functions, exponentials, and powers	876
7.68 Combinations of degenerate hypergeometric functions and	
other special functions	881
7.69 Integration of degenerate hypergeometric functions with	
respect to the index	884
7.7 Parabolic-Cylinder Functions	885
7.71 Parabolic-cylinder functions	885
7.72 Combinations of parabolic-cylinder functions, powers, and	
exponentials	885
7.73 Combinations of parabolic-cylinder and hyperbolic functions	887
7.74 Combinations of parabolic-cylinder and trigonometric func-	
tions	888
7.75 Combinations of parabolic-cylinder and Bessel functions	889
7.76 Combinations of parabolic-cylinder functions and degenerate	004
hypergeometric functions	894
7.77 Integration of a parabolic-cylinder function with respect to	0.05
the index 7.8 Majior's and MacPobert's Functions (C and F)	895
7.8 Meijer's and MacRobert's Functions (G and E)	896
7.81 Combinations of the functions G and E and the elementary	007
functions	896

CONTENTS	xix
7.82 Combinations of the functions G and E and Bessel functions 7.83 Combinations of the functions G and E and other special	900
functions	903
8 9. SPECIAL FUNCTIONS	904
8.1 Elliptic Integrals and Functions	904
8.11 Elliptic integrals	904
8.12 Functional relations between elliptic integrals	907
8.13 Elliptic functions	909
8.14 Jacobian elliptic functions	910
8.15 Properties of Jacobian elliptic functions and functional rela-	
tionships between them	914
8.16 The Weierstrass function $\wp(u)$	917
8.17 The functions $\zeta(u)$ and $\sigma(u)$	920
8.18-8.19 Theta functions	921
8.2 The Exponential-Integral Function and Functions Generated by It	925
8.21 The exponential-integral function $E_i(x)$	925
8.22 The hyperbolic-sine-integral $shix$ and the hyperbolic-cosine-	
integral $chix$	928
8.23 The sine integral and the cosine integral: $si(x)$ and $ci(x)$	928
8.24 The logarithm-integral $li(x)$	929
8.25 The probability integral and Fresnel integrals $\Phi(x)$ , $S(x)$ and	
C(x)	930
8.26 Lobachevskiy's function $L(x)$	933
8.3 Euler's Integrals of the First and Second Kinds and Functions	
Generated by Them	933
8.31 The gamma function (Euler's integral of the second kind):	
$\Gamma(z)$	933
8.32 Representation of the gamma function as series and products	935
8.33 Functional relations involving the gamma function	937
8.34 The logarithm of the gamma function	939
8.35 The incomplete gamma function	940
8.36 The psi function $\psi(x)$	943
8.37 The function $\beta(x)$	947
8.38 The beta function (Euler's integral of the first kind): $B(x, y)$	948
8.4-8.5 Bessel Functions and Functions Associated with Them	951
8.40 Definitions	951
8.41 Integral representations of the functions $J_v(z)$ and $N_v(z)$	952
8.42 Integral representations of the functions $H_v^{(1)}(z)$ and $H_v^{(2)}(z)$	955
8.43 Integral representations of the functions $I_v(z)$ and $K_v(z)$	958
8.44 Series representation	959
8.45 Asymptotic expansions of Bessel functions	961

XX CONTENTS

8.46 Bessel functions of order equal to an integer plus one-half	965
8.47-8.48 Functional relations	967
8.49 Differential equations leading to Bessel functions	971
8.51-8.52 Series of Bessel functions	973
8.53 Expansion in products of Bessel functions	979
8.54 The zeros of Bessel functions	980
8.55 Struve functions	982
8.56 Thomson functions and their generalizations: $ber_v(z)$ , $bei_v(z)$ ,	
$\operatorname{her}_{v}(z)$ , $\operatorname{hei}_{v}(z)$ , $\operatorname{ker}(z)$ , $\operatorname{kei}(z)$	983
8.57 Lommel functions	988
8.58 Anger and Weber functions $J_{v}(z)$ and $E_{v}(z)$	988
8.59 Neumann's and Schläfli's polynomials: $O_n(z)$ and $S_n(z)$	989
8.6 Mathieu Functions	99
8.60 Mathieu's equation	99
8.61 Periodic Mathieu functions	99
8.62 Recursion relations for the coefficients $A_{2r}^{(2n)}$ , $A_{2r+1}^{(2n+1)}$ ,	
$B_{2r+1}^{(2n+1)}, B_{2r+2}^{(2n+2)}$	99
8.63 Mathieu functions with a purely imaginary argument	99
8.64 Nonperiodic solutions of Mathieu's equation	99
8.65 Mathieu functions for negative $q$	998
8.66 Representation of Mathieu functions as series of Bessel	
functions	99
8.67 The general theory	99
8.7-8.8 Associated Legendre Functions	998
8.70 Introduction	99
8.72 Asymptotic series for large values of $ v $	100
8.73-8.74 Functional relations	100
8.75 Special cases and particular values	100
8.76 Derivatives with respect to the order	100
8.77 Series representation	100
8.78 The zeros of associated Legendre functions	101
8.79 Series of associated Legendre functions	101
8.81 Associated Legendre functions with integral indices	101
8.82-8.83 Legendre functions	101
8.84 Conical functions	102
8.85 Toroidal functions	102
8.9 Orthogonal Polynomials	102
8.90 Introduction	102
8.91 Legendre polynomials	102
8.92 Series of Legendre polynomials	102
8.93 Gegenbauer polynomials $C_{\pi}^{\lambda}(t)$	102

CONTENTS	XX	۲i

8.94 The Chebyshev polynomials $T_n(x)$ and $U_n(x)$	1032
8.95 The Hermite polynomials $H_n(x)$	1033
8.96 Jacobi's polynomials	1035
8.97 The Laguerre polynomials	1037
9.1 Hypergeometric Functions	1039
9.10 Definition	1039
9.11 Integral representations	1040
9.12 The representation of elementary functions in terms of a	
hypergeometric function	1040
9.13 Transformation formulas and the analytic continuation of	
functions defined by hypergeometric series	1042
9.14 A generalized hypergeometric series	1045
9.15 The hypergeometric differential equation	1045
9.16 Riemann's differential equation	1049
9.17 Representation of certain second-order differential equations	
by means of a Riemann scheme	1052
9.18 Hypergeometric functions of two variables	1053
9.19 A hypergeometric function of several variables	1057
9.2 A Degenerate Hypergeometric Function	1057
9.20 Introduction	1057
9.21 The functions $\Phi(\alpha, \gamma; z)$ and $\Psi(\alpha, \gamma; z)$	1058
9.22-9.23 The Whittaker functions $M_{\lambda,\mu}(z)$ and $W_{\lambda,\mu}(z)$	1059
9.24-9.25 Parabolic cylinder functions $D_p(z)$	1064
9.26 Degenerate hypergeometric series of two variables	1067
9.3 Meijer's G-Function	1068
9.30 Definition	1068
9.31 Functional relations	1069
9.32 A differential equation for the G-function	1070
9.33 Series of G-functions	1070
9.34 Connections with other special functions	1070
9.4 MacRobert's E-Function	1071
9.41 Representation by means of multiple integrals	1071
9.42 Functional relations	1072
9.5 Riemann's Zeta Functions $\zeta(z,q)$ , and $\zeta(z)$ , and the Functions	
$\Phi(z, s, v)$ and $\xi(s)$	1072
9.51 Definition and integral representations	1072
9.52 Representation as a series or as an infinite product	1073
9.53 Functional relations	1073
9.54 Singular points and zeros	1074
9.55 The function $\Phi(z, s, v)$	1075
9.56 The function $\xi(s)$	1076
•	

•

xxii CONTENTS

9.6 Bernoulli Numbers and Polynomials, Euler Numbers, the	
Functions $v(x)$ , $v(x, a)$ , $\mu(x, \beta)$ , $\mu(x, \beta, a)$ , $\lambda(x, y)$	1076
9.61 Bernoulli numbers	1076
9.62 Bernoulli polynomials	1077
9.63 Euler numbers	1078
9.64 The functions $v(x)$ , $v(x, \alpha)$ , $\mu(x, \beta)$ , $\mu(x, \beta, \alpha)$ , $\lambda(x, y)$	1079
9.7 Constants	1079
9.71 Bernoulli numbers	1079
9.72 Euler numbers	1080
9.73 Euler's and Catalan's constants	1086
BIBLIOGRAPHIC REFERENCES USED IN PREPARATION OF TEXT	108
CLASSIFIED SUPPLEMENTARY REFERENCES	1083