CONTENTS

CHAPTER 1. **Basic theory** 1

1. Introduction 1
2. Basic concepts of abstract graphs 3
   - 2.1. General definitions 3
   - 2.2. Isomorphism 6
   - 2.3. Connectedness 8
   - 2.4. Rank and nullity 11
   - 2.5. Degrees 12
3. Operations on graphs 13
4. Some important classes of graphs 17
   - 4.1. Planar graphs 17
   - 4.2. Separable and nonseparable graphs 19
   - 4.3. Bipartite graphs 22
5. Directed graphs 23
   - 5.1. Basic concepts 24
   - 5.2. Directed-edge sequence 27
   - 5.3. Outgoing and incoming degrees 29
   - 5.4. Strongly-connected directed graphs 30
   - 5.5. Some important classes of directed graphs 31
6. Mixed graphs 32
7. Conclusions 32

Problems 33

CHAPTER 2. **Foundations of electrical network theory** 36

1. Matrices and directed graphs 37
   - 1.1. The node-edge incidence matrix 37
   - 1.2. The circuit-edge incidence matrix 41
   - 1.3. The cut-edge incidence matrix 46
   - 1.4. Interrelationships among the matrices A, B_f, and Q_f 53
   - 1.5. Vector spaces associated with the matrices B_a and Q_a 57
2. The electrical network problem 58
3. Solutions of the electrical network problem 62
   - 3.1. Branch-current and branch-voltage systems of equations 63
   - 3.2. Loop system of equations 63
   - 3.3. Cut system of equations 70
   - 3.4. Additional considerations 76
4. Invariance and mutual relations of network determinants and the generalized cofactors 77

4.1. A brief history 77
4.2. Preliminary considerations 78
4.3. The loop and cut transformations 83
4.4. Network matrices 85
4.5. Generalized cofactors of the elements of the network matrix 95
5. Invariance and the incidence functions 107
6. Topological formulas for *RLC* networks 111
6.1. Network determinants and trees and cotrees 111
6.2. Generalized cofactors and 2-trees and 2-cotrees 114
6.3. Topological formulas for *RLC* two-port networks 122
7. The existence and uniqueness of the network solutions 125
8. Conclusions 132
Problems 133

CHAPTER 3. **Directed-graph solutions of linear algebraic equations** 140
1. The associated Coates graph 141
1.1. Topological evaluation of determinants 142
1.2. Topological evaluation of cofactors 146
1.3. Topological solutions of linear algebraic equations 149
1.4. Equivalence and transformations 155
2. The associated Mason graph 167
2.1. Topological evaluation of determinants 169
2.2. Topological evaluation of cofactors 172
2.3. Topological solutions of linear algebraic equations 174
2.4. Equivalence and transformations 177
3. The modifications of Coates and Mason graphs 189
3.1. Modifications of Coates graphs 189
3.2. Modifications of Mason graphs 197
4. The generation of subgraphs of a directed graph 199
4.1. The generation of 1-factors and 1-factorial connections 201
4.2. The generation of semifactors and k-semifactors 203
5. The eigenvalue problem 206
6. The matrix inversion 210
7. Conclusions 216
Problems 216

CHAPTER 4. **Topological analysis of linear systems** 224
1. The equicofactor matrix 225
2. The associated directed graph 230
2.1. Directed-trees and first-order cofactors 231
2.2. Directed 2-trees and second-order cofactors 244
3. Equivalence and transformations 251
4. The associated directed graph and the Coates graph 262
4.1. Directed trees, 1-factors, and semifactors 262
4.2. Directed 2-trees, 1-factorial connections, and 1-semifactors 266
5. Generation of directed trees and directed 2-trees 269

5.1. Algebraic formulation 269
5.2. Iterative procedure 272
5.3. Partial factoring 279
6. Direct analysis of electrical networks 281
6.1. Open-circuit transfer-impedance and voltage-gain functions 281
6.2. Short-circuit transfer-admittance and current-gain functions 289
6.3. Open-circuit impedance and short-circuit admittance matrices 294
6.4. The physical significance of the associated directed graph 297
6.5. Direct analysis of the associated directed graph 302
7. Conclusions 311
Problems 312

CHAPTER 5. **Trees and their generation** 320
1. The characterizations of a tree 320
2. The codifying of a tree-structure 325
2.1. Codification by paths 326
2.2. Codification by terminal edges 328
3. Decomposition into paths 330
4. The Wang-algebra formulation 332
4.1. The Wang algebra 333
4.2. Linear dependence 334
4.3. Trees and cotrees 338
4.4. Multi-trees and multi-cotrees 340
4.5. Decomposition 345
5. Generation of trees by decomposition without duplications 353
5.1. Essential complementary partitions of a set 353
5.2. Algorithm 356
5.3. Decomposition without duplications 359
6. The matrix formulation 365
6.1. The enumeration of major submatrices of an arbitrary matrix 365
6.2. Trees and cotrees 368
6.3. Directed trees and directed 2-trees 370
7. Elementary transformations 373
8. Hamilton circuits in directed-tree graphs 379
9. Directed trees and directed Euler lines 384
10. Conclusions 389
Problems 390

CHAPTER 6. **The realizability of directed graphs with prescribed degrees** 398
1. Existence and realization as a (p, s)-digraph 398
1.1. Directed graphs and directed bipartite graphs 400
1.2. Existence 401
1.3. A simple algorithm for the realization 413
1.4. Degree invariant transformations 419
1.5. Realizability as a connected (p, s)-digraph 422
2. Realizability as a symmetric (p, s)-digraph 427

2.1. Existence 428
2.2. Realization 433
2.3. Realizability as connected, separable and nonseparable graphs 436
3. Unique realizability of graphs without self-loops 440
3.1. Preliminary considerations 441
3.2. Unique realizability as a connected graph 443
3.3. Unique realizability as a graph 446
4. Existence and realization of a (p, s)-matrix 448
5. Realizability as a weighted directed graph 452
6. Conclusions 454
Problems 455

Bibliography 464

Symbol index 473

Subject index 478